Name: Lesson 8 Solutions Class #:

Math 48A, Lesson 8: Absolute Value Functions

1. THE ABSOLUTE VALUE OPERATION

1A. Suppose you're talking to your abuelita (grandma) and she is not familiar with the idea of an "absolute value". Explain to your abuelita what it means to find the "absolute value" of a number. Put your description into words.

- when we take the absolute value of a number, if the number is negative then the output would be positive —Jaliri
- If we take the absolute value of a positive number, it stays positive
- absolute value represents distance
- To find the output, count how many spots you are from zero...

1B. Show how to calculate the absolute value of at least two numbers (find the absolute value of at least one positive and one negative number)

Let $f(x) = |x|$ and consider

Example 1: $f(4) = |4| = 4$

$f(-6) = |-6| = 6$

$f(0) = |0| = 0$

☐ the absolute value is the representaton of the physical distance from zero.

☐ distance <u>can NOT be negative</u> by the nature of that measurement

☐ Absolute value is always ~~positive~~ nonnegative

(it's always either positive or zero)

☐ the absolute value of zero is zero.

☐ $$f(x) = |x| = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

(input is

absolute value bars

curly bracket

piecewise function notation

Lets explore a bit more:

$$f(x) = |x| = \begin{cases} \text{output} & \text{input information} \\ +1 = -(-1) & \text{if } x = -1 \\ +2 = -(-2) & \text{if } x = -2 \\ +3 = -(-3) & \text{if } x = -3 \\ \boxed{-x} & \text{if } x < 0 \end{cases}$$

this output value turns into a positive

this input value is negative

Famous Trick

sign of one number

sign of another number

	+	−
+	+	−
−	−	+

sign of product

eg.

□ $+1 \cdot +1 = +1 \Leftrightarrow + \cdot + = +$

□ $+1 \cdot -1 = -1 \Leftrightarrow + \cdot - = -$

□ $-1 \cdot +1 = -1 \Leftrightarrow - \cdot + = -$

□ $-1 \cdot -1 = +1 \Leftrightarrow - \cdot - = +$

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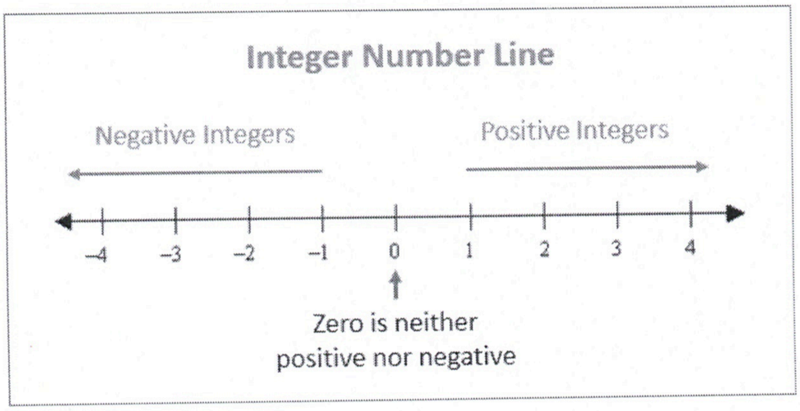
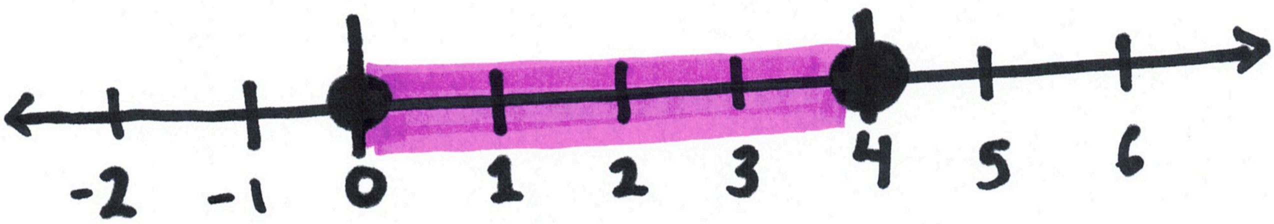
1C. Consider the following diagram of a small section of the real number line:



Use this type of diagram to explain using visual imagery what it means to take the absolute value of a number relates to the real number line. Make sure to demonstrate your ideas using examples of positive AND negative numbers.

Example 1 diagram: $\underbrace{f(4) = |4| = 4}$

the distance between zero and four has a length of 4 units



left absolute value bar

output

$$f(4) = |4| = 4$$

input to function

right absolute value bar

input shows up between the two absolute value bars

Note:

- the output of absolute value can't be negative
- the input can be any real number, positive, zero, or negative
- Notice, suppose I tell you the following statement

left abs value bar

right abs value bar

$$\left| \boxed{\quad} \right| = 4$$

- all things in between the left and right absolute value bars
- the $\boxed{\text{argument}}$ to absolute value
- the $\boxed{\text{expression inside}}$ the absolute value

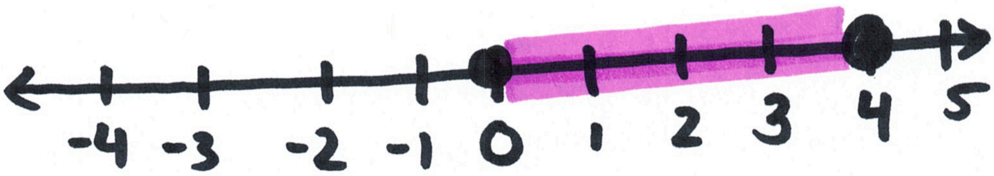
When we say

$$\left| \quad \right| = 4$$

we mean that whatever is inside the absolute value bars is 4 units away from zero:

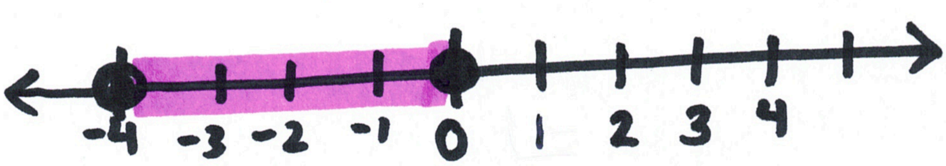
Option 1 for $\quad$ = +4

☐ Set whatever is inside the absolute value equal to the positive of the outp



Option 2 for $\quad$ = -4

☐ Set the argument inside abs value bars equal to the negative of the output



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3. SOLVE AN ABSOLUTE VALUE EQUATION

Use what you learned in problems 1 and 2 above to solve the following absolute value equation. How does your solution relate to the idea of a piecewise function?

$$|3y - 7| - 6 = -2$$

Consider

$$\left| \underbrace{3y - 7}_{\text{argument of the absolute value}} \right| - 6 = -2$$

left abs value bar

right abs value bar

$$|\boxed{3y-7}| - 6 = -2$$

$$\qquad\quad +6 \qquad +6$$

$$\Rightarrow |\boxed{3y-7}| = 4$$

Absolute value of some argument equals four

get rid of absolute value bars

$\Rightarrow$ we have two options for our expression inside the absolve value:

Option 1: $\boxed{3y-7} = +4$

Option 2: $\boxed{3y-7} = -4$

Option 1:

$$3y - \cancel{7} = +4$$

$$\quad +7 \qquad\qquad +7$$

$+0$

$$\frac{\cancel{3} \cdot y}{3} = \frac{11}{3}$$

1

$$\boxed{y = \frac{11}{3}} = 3\,{}^{2}\!/_{3} = 3.66\overline{6}$$

Option 2:

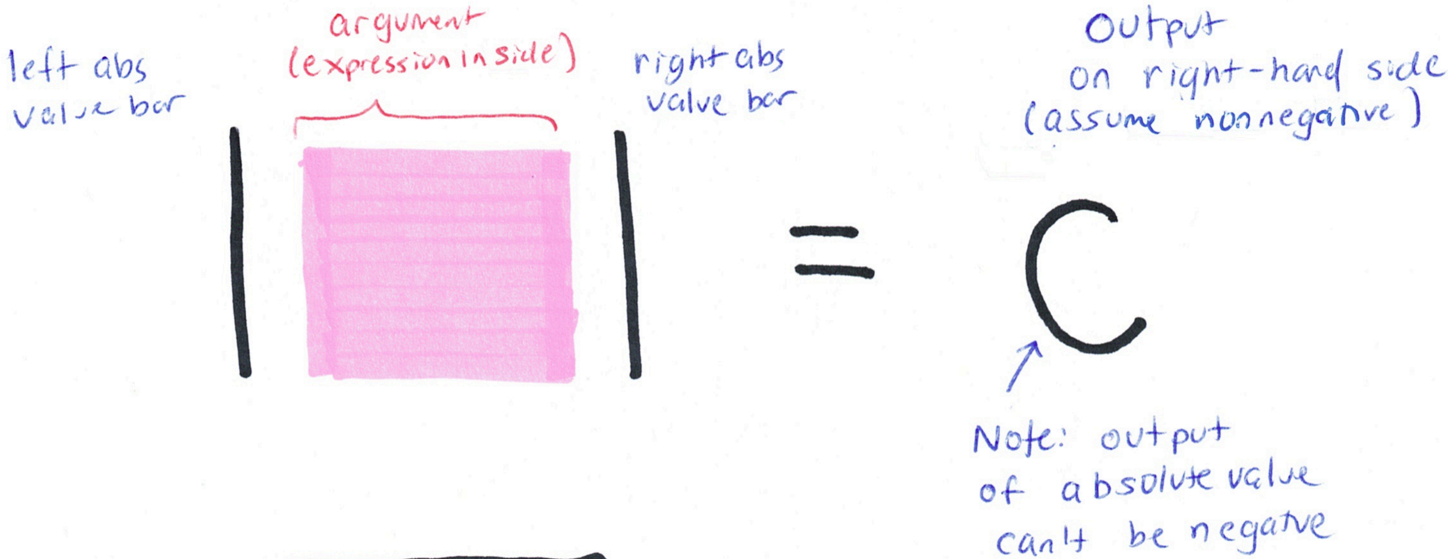
$$3y - 7 = -4$$

$$\quad +7 \qquad\qquad +7$$

$$\frac{3y}{3} = \frac{3}{3}$$

$$\boxed{y = 1}$$

When solving absolute value equations, we work to get out equation in the form



Algebraic Technique

To get rid of absolute value bar, create two new equations:

Equation 1: Set argument equal to positive of right-hand side output

Equation 2: Set expression inside abs value to negative of right-hand side

Let's look at

Piecewise notation

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

rules for output (range information) | Rule for input (Domain info)

rules for calculating output | corresponding rules for the input variable

Eg: $x = -4 \Rightarrow x < 0$

$\Rightarrow f(-4) = f(x) = -x$

$= -(-4)$

$= +4$

Eg: $x = 0 \Rightarrow x = 0 \Rightarrow f(0) = 0$

Eg: $x = 1 \Rightarrow x > 1$

$\Rightarrow f(1) = f(x) = x$

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2. GRAPH THE ABSOLUTE VALUE FUNCTION

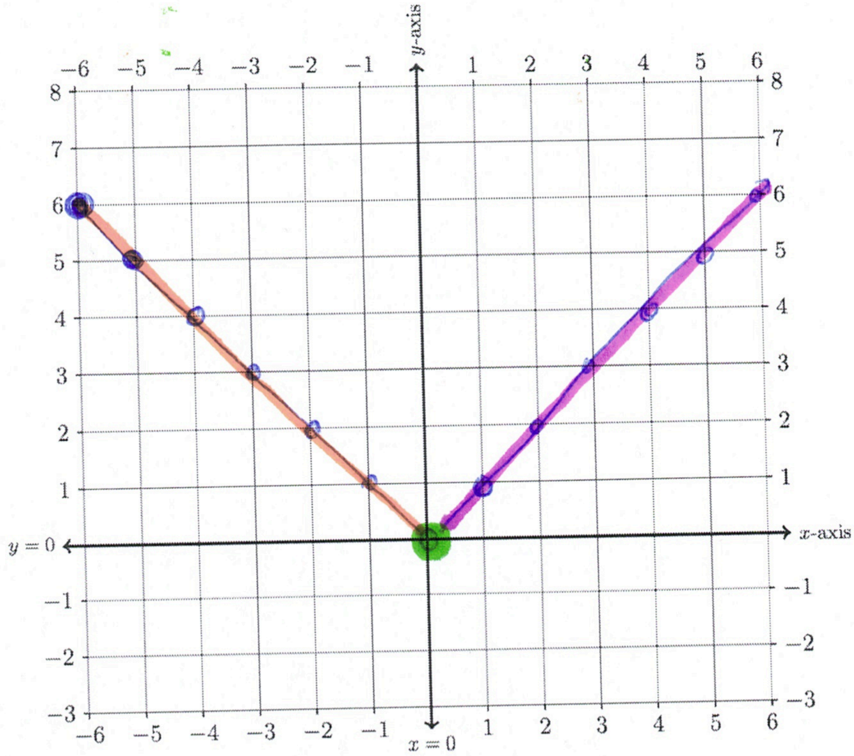
2A. Consider the following piecewise function:

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

Create a table of values and graph the resulting lines on these axes below.

input

x	$f(x)$
-6	$f(-6) = -(-6) = 6$
-5	$f(-5) = -(-5) = 5$
-4	4
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3
4	4
5	5
6	6



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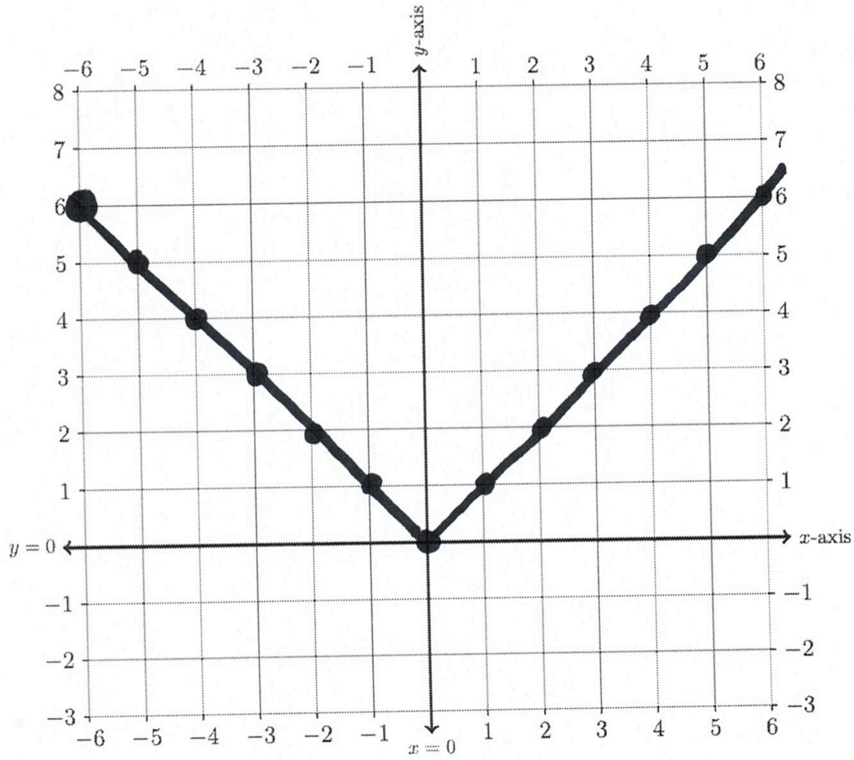
2B. Consider the following absolute value function:

$$f(x) = |\,x\,|$$

Create a table of values and draw the resulting graph on the axes below.

x	$f(x)$
-6	6
-5	5
-4	4
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3
4	4
5	5
6	6



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2C. Look back at your graphs from problem 3A and problem 3B. What do you notice about those two graphs? Make a conjecture (a mathematical guess) that captures your observations.

- We noticed in problems 2A and 2B we have the same graph and table

$$\Rightarrow \quad f(x) = |x| = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

equivalent piecewise definition for absolute value function

2D. What does it mean to discover an equivalent representation for a function?

- Expressing same idea in a different way
- Capture mathematical properties in a different form...
- Analogus to what we did in algebra

$$1 = \frac{2}{2} = 3-2 = 1^2$$

Other interesting facts about abs valve:

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x \leq 0 \end{cases}$$

$$= \sqrt[2]{x^2}$$

x	$\sqrt[2]{x^2}$
3	$\sqrt[2]{3^2} = \sqrt{9} = 3$
2	$\sqrt{2^2} = \sqrt{4} = 2$
1	1
0	0
-1	
-2	
-3	$\sqrt[2]{(-3)^2} = \sqrt{9} = 3$

$$= \operatorname{sgn}(x) \cdot x$$

$$\operatorname{sgn}(x) = \begin{cases} +1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

sign function

or signum function

Eg : $x = 7 \Rightarrow \operatorname{sgn}(x) = \operatorname{sgn}(7) = +1$

$x = -3 \Rightarrow \operatorname{sgn}(x) =$