Name: Lesson 7 Solutions Part 1

Class #:

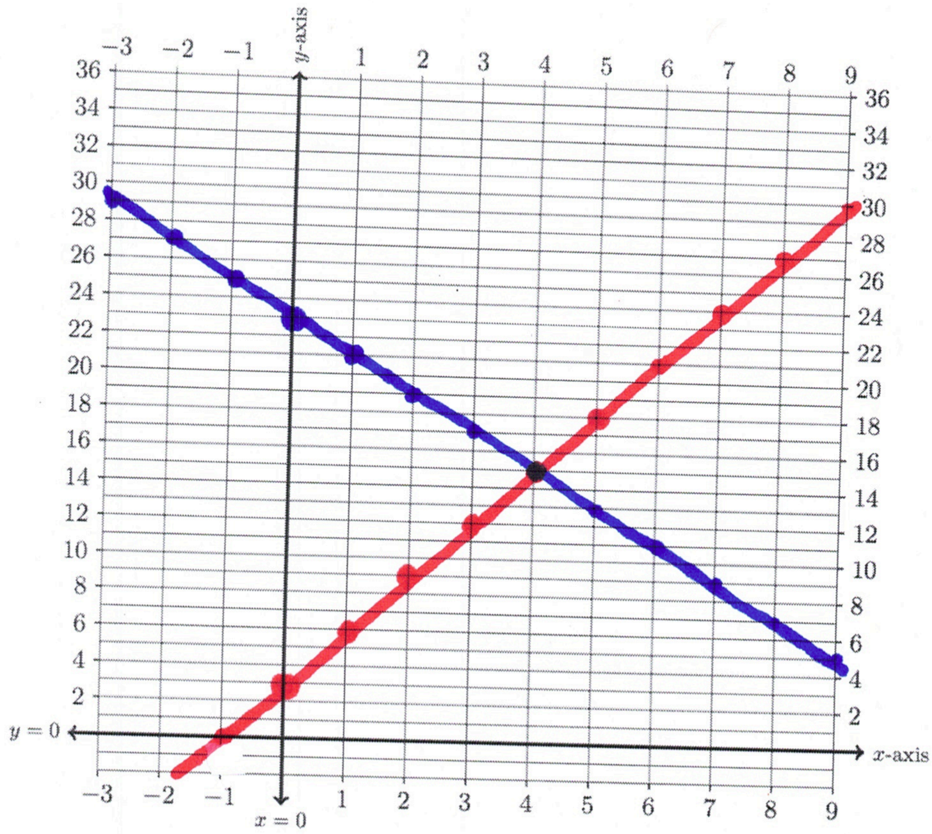
Math 48A, Lesson 7: Linear Functions

1. LINEAR FUNCTIONS HAVE CONSTANT RATES OF CHANGE

1A. Consider the linear functions $\boxed{f(x) = -2x + 23}$ and $\boxed{g(x) = 3x + 3}$

Create a table of values and graph the resulting lines on these axes below.

x	$f(x)$	$g(x)$
-4	31	-9
-3	29	-6
-2	27	-3
-1	25	0
0	23	3
1	21	6
2	19	9
3	17	12
4	15	15
5	13	18
6	11	21
7	9	24
8	7	27
9	5	30



Note:

$f(x) = -2x + 23$

slope   y-intercepts

$g(x) = 3x + 3$

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1B. Which function is increasing? Which function is decreasing? Explain why.

Remember from pgs 10 - 13 from Lesson 5, we said:

- ☐ The $f(x) = -2x + 23$ function is decreasing because as the x-values increase the y-values decrease at a constant rate

as we move right in input

we move down in output

- ☐ If $x_1 < x_2$ and $f(x_1) > f(x_2)$

(as we move from smaller to bigger in x-values)

(we move from bigger to smaller in y-values)

and we say f(x) is decreasing

- [ ] The $g(x) = 3x + 3$ is increasing because the slope is positive ...

- [ ] Another way to say this is:

we move up in output

as we move right in input

If $x_1 < x_2$ and $f(x_1) < f(x_2)$

(as we move from smaller to bigger in x-values move left to right on graph)

(we also move from smaller to bigger In te y-value)

then the function is increasing

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Calculate the average rate of change of $f(x)$ and $g(x)$ between the points

1C. between $x = 1$ and $x = 6$. (on an interval $[1, 6]$)

1D. between $x = a$ and $x = a + h$.

How are your answers related to your work on problem 1B?

Claim: When studying lines, we get an idea about increasing and decreasing from slopes.

Recall: ▫ a secant line is a line through two points on a graph

Point 1: $(x_1, f(x_1))$

Point 2: $(x_2, f(x_2))$

▫ Average rate of change is slope of the secant line

4

with equation

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Super helpful in math 1A & math 12

$$= \boxed{\frac{f(x_2) - f(x_1)}{x_2 - x_1}}$$

$$= \frac{\Delta y}{\Delta x}$$

delta y over delta x

$$= \frac{\text{change in } y}{\text{change in } x}$$

1c. ARoC of $f(x) = -2x + 23$

between $x = 1$ and $x = 6$:

To calcate ARoC, we need

two points:

Point 1: $(1, f(1)) = (1, 21)$

Point 2: $(6, f(6)) = (6, 11)$

$$\Rightarrow \text{ARoC} = m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{11 - 21}{6 - 1}$$

$$= \frac{-10}{5} = \boxed{-2} \quad \boxed{\begin{array}{l}\text{decreasing} \\ < 0\end{array}}$$

slope of line

6

★ Remember for future calc classes

10. AROC of $f(x) = -2x + 23$

between $x_1 = a$ and $x_2 = a+h$

To calculate an AROC we need two points:

Point 1: $(x_1, f(x_1)) = (a, -2a + 23)$

If $x_1 = a$ then

$f(x_1) = f(a)$

$= -2x + 23 \Big|_{x=a}$

$= -2a + 23 = y_1$

Point 2 : $(x_2, f(x_2)) = (a+h, f(a+h))$

$=$

If $x_2 = a+h$ then

$$f(x_2) = f(a+h)$$

$$= -2x + 23 \Big|_{x=a+h}$$

distribute

$$= -2\cdot(a+h) + 23$$

$$= -2a + -2\cdot h + 23 = y_2$$

To find ARoC, let's calculate

$$\text{ARoC} = m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{\overbrace{(-2a + -2h + 23)}^{f(x_2)} - \overbrace{(-2a + 23)}^{f(x_1)}}{a+h - a}$$

distribute

Don't do dumb stuff

do no more than two algebra steps per line

$$= \frac{\boxed{\cancel{-2a}} - 2h \boxed{\cancel{+23}} \boxed{\cancel{+2a}} \boxed{\cancel{-23}}}{h}$$

$$= \frac{-2 \cdot \boxed{\cancel{h}}}{\boxed{\cancel{h}}} = -2 \boxed{< 0 \quad \text{decreasing}}$$

9

$= \dfrac{-2 \cdot h}{1 \cdot h} \quad \leftarrow \quad h = 1 \cdot h$

because when there's no coefficient in front of a variable, there's always like a hidden 1

multiply across

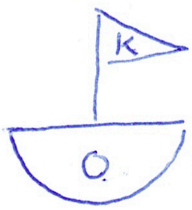
$= \dfrac{-2}{1} \cdot \dfrac{h}{h} \quad \ldots\ldots \leftarrow \quad \dfrac{A \cdot C}{B \cdot D} = \dfrac{A}{B} \cdot \dfrac{C}{D}$

break 'um up

$\dfrac{A}{B}$

$= \dfrac{-2}{1} \cdot 1 \quad \leftarrow \quad \dfrac{h}{h} = 1 \text{ if } h \neq 0$

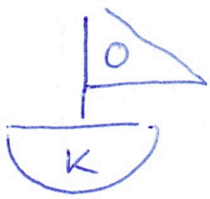
- we never divide by zero



← when hole in boat, we get KO'd

$= -2 \cdot 1$

- we say $0/K = 0$ when $K \neq 0$



← we're OK

$= \boxed{-2}$

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2. GENERAL FORMS FOR A LINE

Homework

Write your 1st draft of your description for three different forms for the equation of a line including:

y-intercept at point (0,b)

Slope-intercept form: $y = m\,x + b$

slope

Jeff's favorite → Point-slope form: $y - y_1 = m\,(x - x_1)$ ← (x_1, y_1) is a point on our line

Polynomial form: $y = a_1 x + a_0$

As you craft your descriptions of these equations, be sure to use both:

Abuelita language: Use language that your abuelita can understand.

Nerdy language: Write this out using nerdy language. See if you can include formal mathematical symbols. This is the formal concept definition found in your textbook.

Math learning principle:

- ☐ Do strategic hard work
- ☐ Work really hard in smart ways
- ☐ Memorize one formula and get other formulas for free: two-for-one deal

To create an equation for a line in point - slope form, I need

a slope m

and a one point (x_1, y_1)

Then equation for that line is

$$y - y_1 = m \cdot (x - x_1)$$

$$\Rightarrow \quad m = \frac{y - y_1}{x - x_1}$$

equation for slope

y-intercept

$\Rightarrow$ let point $(x_1, y_1) = (0, b)$

$$\Rightarrow \quad y - b = m \cdot (x - 0)$$

$$\Rightarrow \quad y - b = m \cdot x$$

$$\Rightarrow \quad \boxed{y = \underset{\text{slope}}{m}\,x + \underset{\text{y-intercept}}{b}}$$

slope - intercept form

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4. IDENTIFY LINEAR FUNCTIONS

Determine if the function given in each problem below is linear or not. If the function is linear, express this function in slope-intercept form.

4A. $f(x) = \frac{2x-3}{x}$

4B. $\boxed{f(x) = \frac{1}{4}(2-3x) \text{ : linear}}$

4C. $f(x) = \frac{4-3x}{5}$

4D. $f(x) = (x-3)^2$

Let's start with 4B.

distrivity

Consider $f(x) = \frac{1}{4} \cdot (2 - 3x)$

$$= \frac{1}{4} \cdot \boxed{2} - \frac{1}{4} \cdot 3x$$

$$= \frac{1}{4} \cdot \frac{2}{1} - \frac{1}{4} \cdot \frac{3x}{1}$$

multiply across

Note: $\frac{A}{B} \cdot \frac{C}{D} = \frac{A \cdot C}{B \cdot D}$

$$= \frac{1 \cdot 2}{4 \cdot 1} - \frac{1 \cdot 3x}{4 \cdot 1}$$

$$= \frac{2}{4} - \frac{3x}{4}$$

$$= \frac{2 \cdot 1}{2 \cdot 2} - \frac{3x}{4}$$

$$= \boxed{\frac{2}{2}} \cdot \frac{1}{2} - \frac{3x}{4}$$

$$= \boxed{1} \cdot \frac{1}{2} - \frac{3x}{4}$$

$$= \frac{1}{2} \boxed{-} \frac{3x}{4}$$

<u>general form</u>

$y = mx + b$

$$= \frac{1}{2} \boxed{+ \quad -} \frac{3x}{4}$$

Remember

$$A - B = A + -B$$

"A minus B equals A plus negative B"

$$= -\frac{3x}{4} + \frac{1}{2}$$

Remember

$$A + B = B + A$$

$$= -\frac{3 \cdot x}{4 \cdot 1} + \frac{1}{2}$$

$$= -\frac{3}{4} \cdot \frac{x}{1} + \frac{1}{2}$$

$$= \boxed{-\frac{3}{4}} \cdot x + \boxed{\frac{1}{2}}$$

$$= \boxed{m} \; x + \boxed{b}$$

w/ $m = \frac{-3}{4}$

$b = \frac{1}{2}$