Name: In-Class Notes: Lesson 6 Class #:______

Math 48A, Lesson 6: Average Rate of Change

1. SOLVE RADICAL EQUATIONS USING GRAPHS

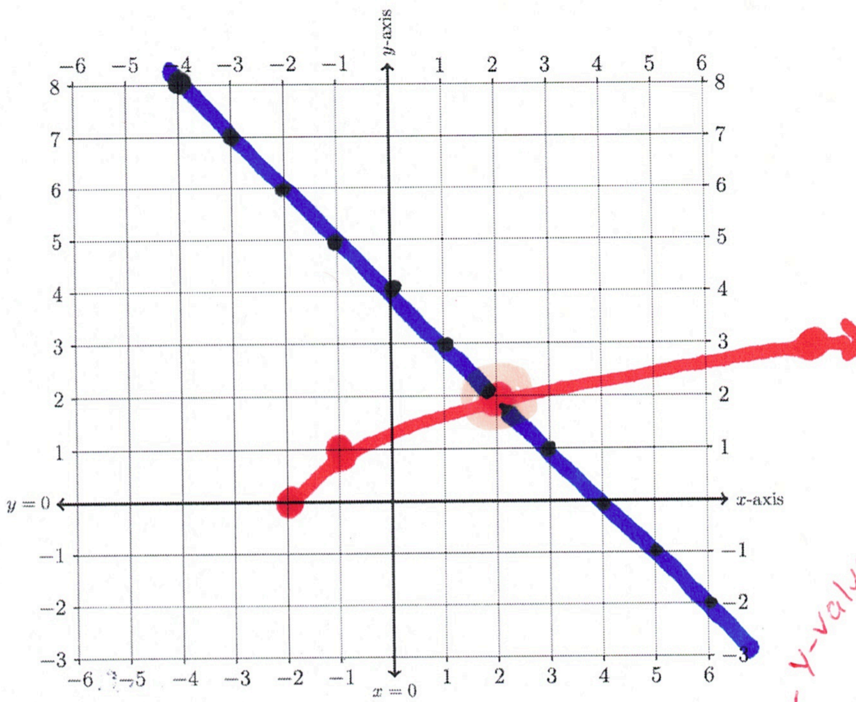
1A. Consider the radical equation below:

$$\boxed{\sqrt[2]{x+2}} = \boxed{4 - x}$$

LHS RHS

Create a table of values and graph the resulting curves on these axes below. Using that work, solve each of the following problems. For each problem, graph the solution interval on the axis provided. Make explicit connections between your solution and the graphs that you draw.

x	Left-hand side: $\sqrt[2]{x+2}$	Right-hand side: $4-x$
−4	Error	8
−3	undefined	7
−2	0	6
−1	1	5
0	1.4142...	4
1	1.7320...	3
2	2	2
3	2.2306...	1
4		0
5		−1



y-value

Point of intersection: (2, 2)

x-value

$\Rightarrow$ $x = 2$ solves this equation

Let $x = -4$ and $f(x) = \sqrt[2]{x+2}$

evaluation bar

$$\Rightarrow f(-4) = \sqrt[2]{x+2}\,\Big|_{x=-4}$$

input

$$= \sqrt[2]{-4+2}$$

$$= \sqrt[2]{-2} \quad \text{Not possible}$$

Note: Remeber the following

Radical Notation		Power Notation
$\sqrt[2]{4} = 2$	$\Leftrightarrow$	$4 = 2^2$
$\sqrt[2]{9} = 3$	$\Leftrightarrow$	$9 = 3^2$
$\sqrt[2]{36} = 6$	$\Leftrightarrow$	$36 = 6^2$

$$\sqrt[2]{a} = b \iff a = b^2$$

" the square root of a equals be is equivalent to finding b such that a equals b squared"

$$\sqrt[2]{-2} = b \iff -2 = b^2$$

□ imaginary number (complex numbers)

Note about b^2

	sign of b	sign of b^2	
positive	+	+	positive
zero	0	0	zero
negative	−	+	positive

Sign chart :

	+	−
+	+	−
−	−	+

Notice that the number b^2 can NOT be negative no matter what b we start with.

Not possible

$$\Rightarrow \quad \underbrace{-2}_{} \neq \underbrace{b^2}_{}$$

this is negative

- either positive or zero (nonnegative)
- for sure, can't be negative

General rule in mathematical calculations:

> We can not take the square root of a <u>negative</u>:

$\sqrt[2]{-2}$ ▫ Does not exist (DNE) as a real number

$\sqrt[2]{-3}$ ▫ D.N.E.

$$\boxed{\text{If } a < 0, \text{ then } \sqrt[2]{a} \text{ does not exist.}}$$

Let $x = -3$ and $f(x) = \sqrt[2]{x+2}$

$$\Rightarrow f(-3) = \sqrt[3]{x+2}\,\Big|_{x=-3}$$

$$= \sqrt[2]{-3+2}$$

$$= \sqrt[2]{-1}$$

we can't take sqrt of a negative

Does not exist, error, not a real number

$x = -2$ and $f(x) = \sqrt[2]{x+2}$

$$\Rightarrow f(-2) = \sqrt[2]{-2+2}$$

$$= \sqrt[2]{0}$$

$$= 0$$

$x = -1$ and consider

$$f(-1) = \sqrt[2]{-1 + 2}$$

$$= \sqrt[2]{1}$$

$$= 1$$

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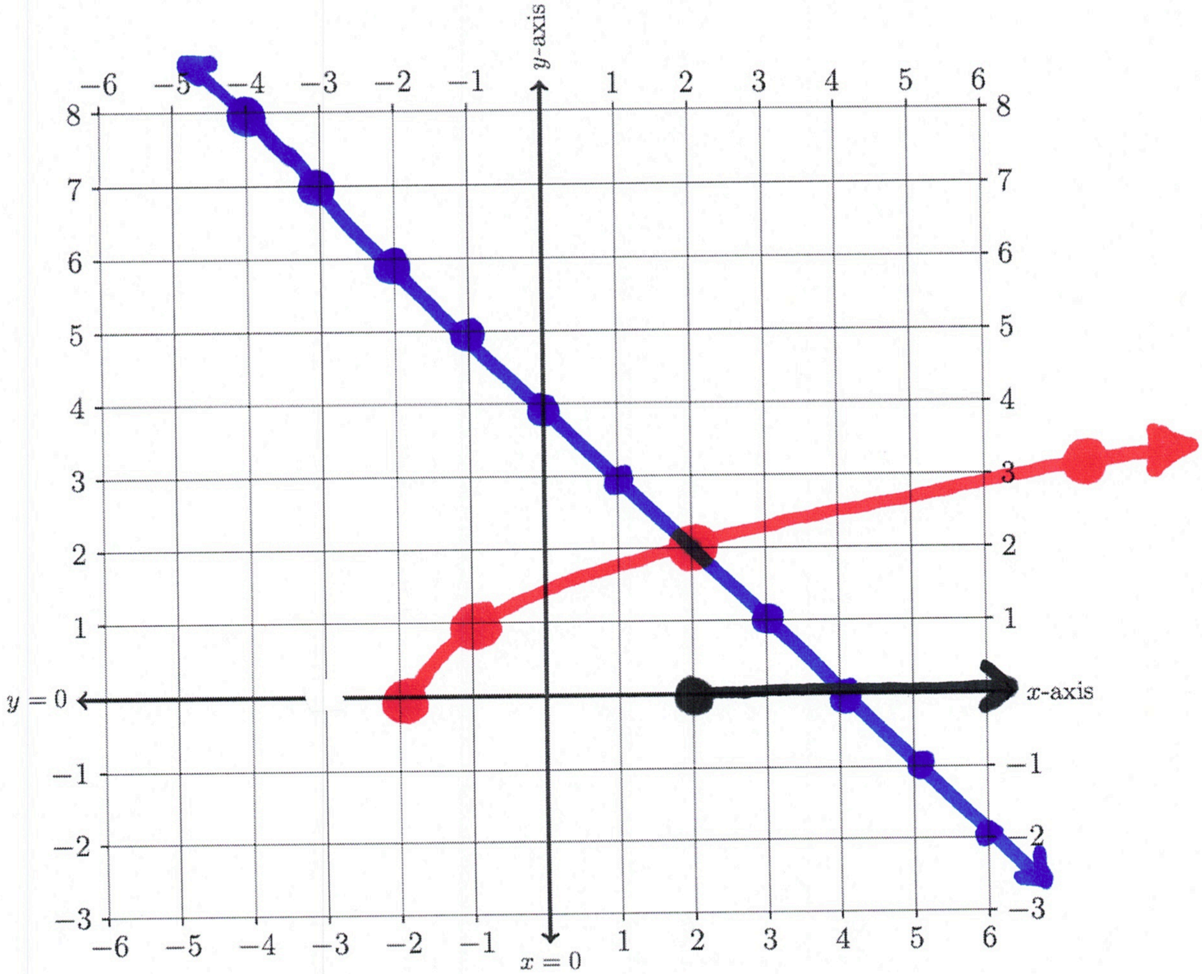
1B. Find all x-values such that: $\boxed{\sqrt[2]{x+2}} \geq \boxed{4-x}$



Nerdy language: "where is left-hand side greater than or equal to right-hand side"

Abuelita language: "where is red curve above or touching the blue line"

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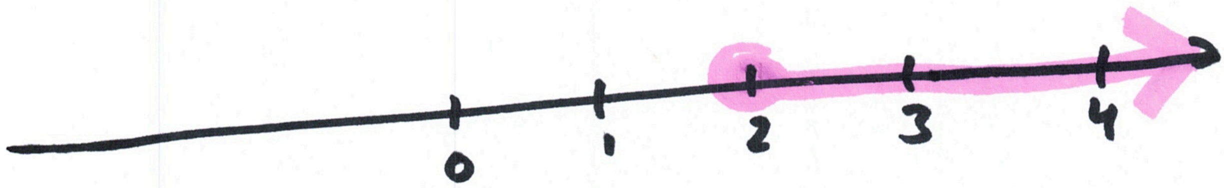
closed bracket

open parentnesis

$$[\,2\,,\ +\infty\,)$$

On the interval $x \in [4, +\infty)$ we solve this inequality.

" x can be greater than or equal to two all the way up to positive infinity"



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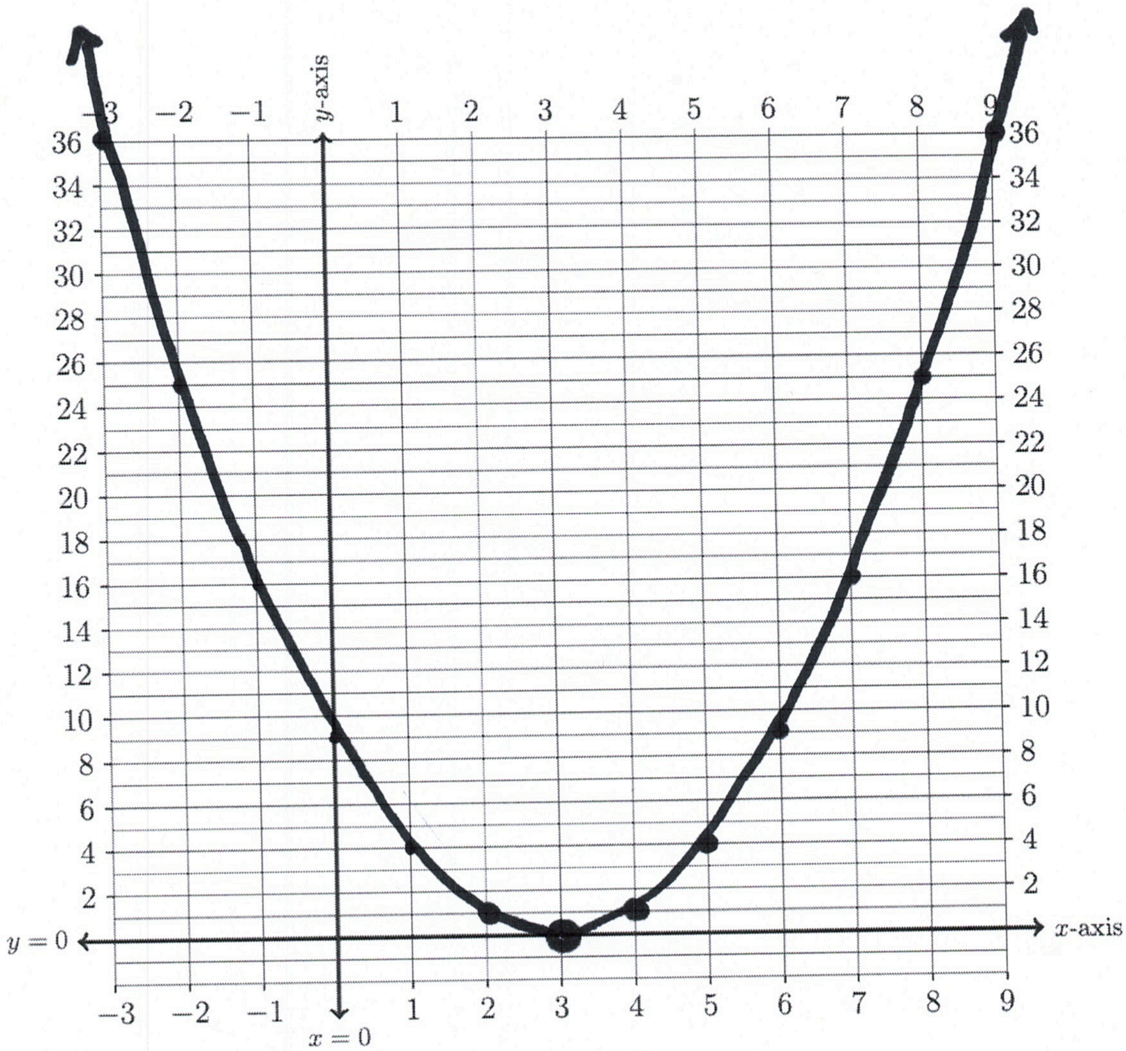
IMPORTANT FOR MATH 12 & MATH 48B

2. AVERAGE RATES OF CHANGE ON A QUADRATIC FUNCTION

2A. Use the table and axes provided below to graph the quadratic function

$$f(x) = (x-3)^2$$

x	$f(x)$
−4	49
−3	36
−2	25
−1	16
0	9
1	4
2	1
3	0
4	1
5	4
6	9
7	16
8	25
9	36



Review for Lesson 5:

☐ This function decreasing as it goes down : $(-\infty, 3)$ open paranthes

☐ At the point $(3, 0)$, we have a local minimum value of $f(x)$

☐ This function is increasing as it goes up on interval: $(3, +\infty)$

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□ A $\boxed{\text{secant line}}$ is a line through two points on a graph

□ Let's remember general formula for a line:

$$y = mx + b$$

slope

y - intercept at point $(0, b)$

$$\text{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\Delta Y}{\Delta X}$$

$$= \frac{\text{change in } y}{\text{change in } x}$$

To calculate slope of a line through two points (x_1, y_1) and (x_2, y_2)

$(a, f(a))$ and $(b, f(b))$

So for our points:

$$(a, f(a)) = (1, f(1))$$
$$(b, f(b)) = (3, f(3))$$

the slope of a secant line

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

$$= \boxed{\frac{f(b) - f(a)}{b - a}}$$

$$= \frac{f(3) - f(1)}{3 - 1}$$

$$= \frac{0 - 4}{3 - 1}$$

Average rate of change

$$= \frac{-4}{2} = \boxed{-2}$$

slope of secant line through point (1, f(1)), (3, f(3

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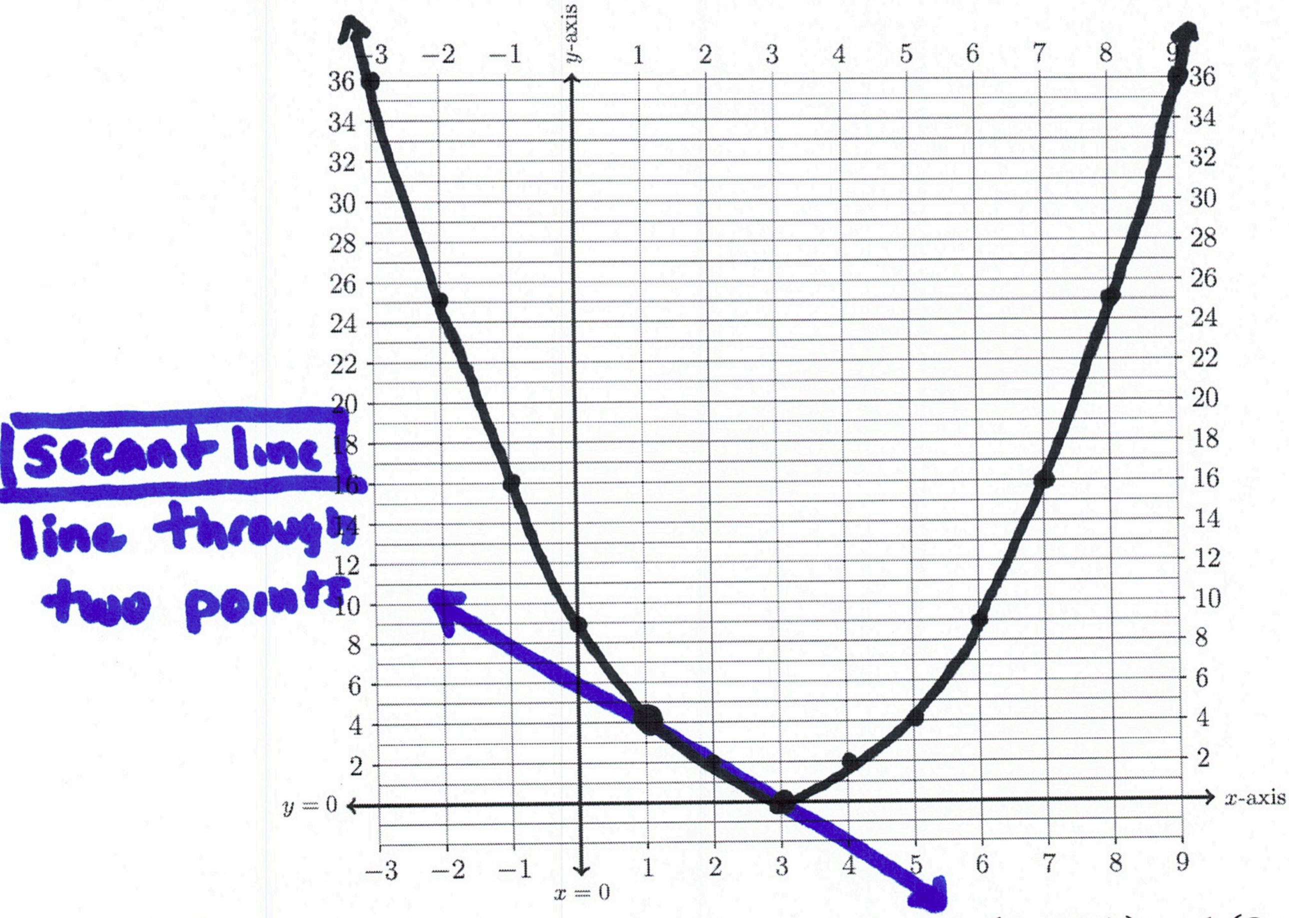
2B. Use the work you did in Problem 2A and re-draw your graph below.



2C. Draw a line between the two points on the graph $(1, f(1))$ and $(3, f(3))$.

2D. Find the slope of the line between points $(1, f(1))$ and $(3, f(3))$.

Let's consider two points on this curve

Point 1: $(1, f(1)) = (1, 4)$ since $f(1) = 4$

Point 2: $(3, f(3)) = (3, 0)$ since $f(3) = 0$

Let's draw a line through two points

We can write

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x_1}$$

$$= \boxed{\frac{f(b) - f(a)}{b - a}} = \frac{\text{rise}}{\text{run}}$$

this is ~~the~~ slope of a secant line through $(a, f(a))$ and $(b, f(b))$

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2E. Write your first draft of a definition for what it means to calculate ***an average rate of change*** of a function $y = f(x)$ between $x = a$ and $x = b$. Include:

Abuelita language: Use language that your abuelita can understand.

Nerdy language: Write this out using nerdy language. See if you can include formal mathematical symbols. This is the formal concept definition found in your textbook.

To calculate average rate of change,

we find slope of our secant line

through two points $(a, f(a))$ and $(b, f(b))$

$$\boxed{m = \frac{f(b) - f(a)}{b - a}} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

Average rate of change : change in output y-values over change in input x-values

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