

Math 48A, Lesson 15: Quadratic Functions

1. Find the Standard Form of a Quadratic Function

Consider the standard form for a quadratic function:

$$f(x) = a x^2 + b x + c$$

Put each of functions below in standard form. In other words, specifically identify the values of a , b , and c .

1A. $f(x) = 5 x^2 - 30 x + 49$

1D. $j(w) = -5 + 3w$

1B. $g(x) = -x^2 + x - 2$

1E. $k(t) = t^2 + 4t$

1C. $h(y) = 4 - y^2$

Name: _____

Class #: _____

2. Explore the Standard Form of a Quadratic Function

Consider the standard form for a quadratic function:

$$f(x) = a x^2 + b x + c$$

Using simple language, identify the role of each individual part of this function. Do your best to come up with descriptions for each of the following: x , x^2 , a , b , and c .

3. Explore the anatomy of perfect-square trinomials

Expand each of the following perfect squares and write as a trinomial in the form $x^2 + b x + c$.

Show your steps and specifically identify values for coefficients b and c . The first one is done for you.

3A. $(x - 4)^2$

Let's consider the perfect square $(x - 4)^2$:

$$(x - 4)^2 = (x - 4) \cdot (x - 4)$$

$$= x \cdot (x - 4) - 4 \cdot (x - 4)$$

$$= x^2 - 4x - 4x + 16$$

$$= x^2 - 8x + 16 \qquad b = -8, c = 16$$

3B. $(x + 3)^2$

Name: _____

Class #: _____

3C. $(x + 11)^2$

3D. $\left(x - \frac{7}{2}\right)^2$

4. For each of the problems above, write the equivalent expressions in the form

$$(x + d)^2 = x^2 + bx + c$$

Then, specifically identify the values of the coefficients d , b and c . The first one is done for you.

4A. $(x - 4)^2$

We notice from our work on problem 3A above that we have:

$$(x - 4)^2 = x^2 - 8x + 16 \qquad d = -4, b = -8, c = 16$$

4B. $(x + 3)^2$

4C. $(x + 11)^2$

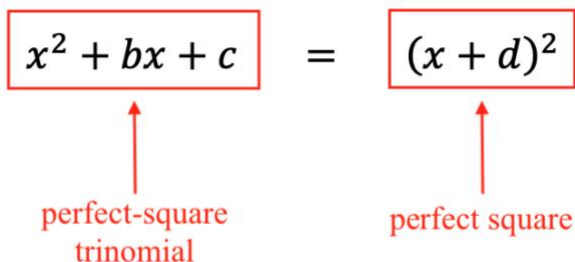
4D. $\left(x - \frac{7}{2}\right)^2$

5. Look back on the work you finished in problem 4 above. What pattern do you notice? Specifically, how are the coefficients d , b and c related to each other? Make a conjecture about how this will work in general.

6. Your definition of a perfect square trinomial
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We say that a perfect-square trinomial is a three-term expression that can be factored as a perfect square. We've seen this in our work in problems 1 – 5. Below is a diagram that shows the connection:

$$\boxed{x^2 + bx + c} = \boxed{(x + d)^2}$$



perfect-square trinomial perfect square

Come up with your own description for a perfect-square trinomial. Use simple, abuelita language and make this as clear as you can for yourself.

7. LEARN TO COMPLETE THE SQUARE

Consider each incomplete expression below. Add a constant to make the expression a perfect-square trinomial. Then write the factored form of the expression as a perfect square. Identify each step you take in the solution. Please make sure you can explain to yourself why you are taking each step.

7A. $x^2 + 10x$

7B. $t^2 - 7t$

Name: _____

Class #: _____

$$7C. \quad x^2 - \frac{11}{2}x$$

$$7D. \quad m^2 + \frac{5}{4}m$$

Name: _____

Class #: _____

7E. $5x^2 - 30x$

8. GENERATE THE VERTEX FORM FOR A QUADRATIC FUNCTION

Use the method of completing the square (from problems 1 – 7) to transform the quadratic function in standard form into an expression that contains a perfect square

8A. $f(x) = 5x^2 + 8x + 3$

8B. $f(x) = ax^2 + bx + c$