Name: Solutions, part 1 Class #:______

Math 48A, Lesson 13: Inverse Functions

1. MORE PRACTICE COMBINING FUNCTIONS

Consider the following functions

$f(x) = x - 3$ and $g(x) = x^2 - 9$

Please find each of the following:

✓ 1A. $(g - f)(x)$

– 1B. $(f \div g)(x) = \frac{f}{g}(x)$

1C. $f \circ g(x) = f(g(x))$

Homework

1D. $g \circ f(x) = g(f(x))$

Homework

Show your work and simplify whenever you can.

Problem 1A: Step 1 Understand the Problem

$$(g-f)(x) = g(x) - f(x)$$

$$= x^2 - 9 - (x - 3)$$

$$= x^2 - 9 - x + 3$$

$$= x^2 \boxed{-x} - 6$$

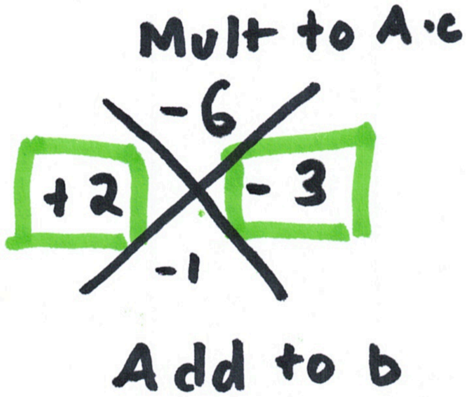
$ax^2 + bx + c$ standard form of a quadratic function

$$= 1x^2 + -1x + -6$$

$(a=1)$ $(b=-1)$ $(c=-6)$

Factor by AC method

Step 1: Form the AC Diamond



Conjectures

NO $-6 = -1 \cdot 6$ $\quad -1 + 6 \neq +5$

$1 + -6 \neq 5$

$-6 = -2 \cdot 3$ $\quad -2 + 3 \neq 1$

$\boxed{2 + -3 = -1}$

Step 2: Split the middle term

$$-1 = +2 - 3$$

$$\Rightarrow -1 \cdot x = (2 - 3) \cdot x$$

$$\Rightarrow -x = \boxed{2 \cdot x - 3 \cdot x}$$

$$x^2 \boxed{-x} - 6$$

$$= \boxed{x^2 + 2x} \boxed{-3x - 6}$$

Step 3: Factor by grouping

$$= x \cdot \boxed{(x+2)} - 3 \cdot \boxed{(x+2)}$$

$$= (x-3)\boxed{(x+2)}$$

$$\Rightarrow \quad x^2 - x - 6 = (x-3) \cdot (x+2)$$

$$\Rightarrow \quad \boxed{(g-f)(x) = (x-3)(x+2)}$$

$$= x^2 - x - 6$$

Problem 1B

$$(f \div g)(x) = \frac{f(x)}{g(x)}$$

$$= \frac{x-3}{x^2-9}$$

difference of squares

$$\square \quad x^2 - a^2 = (x-a)(x+a)$$

$$= x^2 - \cancel{ax} \cancel{+ax} - a^2$$

$$= x^2 - a^2$$

$$\square \quad x^2 - a^2 = (x-a)(x+a)$$

$$= \frac{(x-3)\cdot 1}{(x-3)\cdot(x+3)}$$

Note:

$$\frac{A \cdot C}{B \cdot D} = \frac{A}{B}\cdot\frac{C}{D}$$

$$= \frac{(x-3)}{(x-3)} \cdot \frac{1}{(x+3)}$$

$$\frac{A}{A} = 1 \text{ if } A \neq 0$$

$$= 1 \cdot \frac{1}{x+3} \quad \text{if} \quad x-3 \neq 0$$

$$= \boxed{\frac{1}{x+3}} \quad \text{if} \quad x \neq 3$$

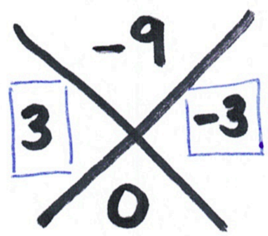
$a=1$, $b=0$, $c=-9$

$$x^2 - 9 = 1x^2 + 0x + -9$$

$ax^2 + bx + c$ standard form

Step 1: Form the AC Diamond

Mult to AC



Add to B

$$\Rightarrow -9 = -1 \cdot 9 \quad \text{NO}$$

$$= -3 \cdot 3$$

$$-3 + 3 = 0$$

Step 2: Split the middle term

$$0 = 3 - 3$$

$$\Rightarrow 0 \cdot x = (3 - 3) \cdot x$$

$$\Rightarrow \boxed{0} = 3x - 3x$$

$$\Rightarrow x^2 - \boxed{0x} - 9$$

$$= \boxed{x^2 + 3x} - \boxed{3x - 9}$$

Group 1 Group 2

Step 3: Factor by grouping

$$= x \cdot \boxed{(x+3)} - 3 \cdot \boxed{(x+3)}$$

$$= (x-3)(x+3)$$

$$\Rightarrow x^2 - 9 = x^2 - 3^2$$

$$= (x-3) \cdot (x+3)$$

Math 12 Tattoo?

$x^2 - a^2 = (x-a)(x+a)$

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2. EXAMPLES OF FORWARD AND BACKWARD PROBLEMS

For each function below, consider solve each of the pairs of problems. As you solve each problem, specifically identify:

A. What is the input and what is the output?
B. What are the known values?
C. What are the unknown values?
D. How many solutions are there?
E. How are the forward and backward problems related?

FUNCTION	FORWARD PROBLEM	BACKWARD PROBLEM
$f(x) = 5 + x$	$5 + 7 = y$	$5 + x = 12$
$g(x) = 4 \cdot x$	$4 \cdot 5 = y$	$4 \cdot x = 20$
$h(x) = x^2$	$9^2 = y$	$x^2 = 81$
$j(x) = x^3$	$(-3)^3 = y$	$x^3 = -27$
$k(x) = \|x\|$	$\|-5\| = y$	$\|x\| = 5$
$m(x) = 2^x$	$2^6 = y$	$2^x = 64$

Let's color code our work:

Black be pre-defined

Blue be "given" as a known input

Red be unknown and desired output

Let's look at function $f(x)$:

$$f(x) = 5 + x$$

Forward problem: Given input $x = 7$

$$\Rightarrow \quad 5 + \underbrace{\boxed{7}}_{\text{given input}} = \underbrace{\boxed{y}}_{\text{unknown output}}$$

function evaluation

Backward problem:

$$\Rightarrow \quad 5 + \underbrace{\boxed{x}}_{\text{unknown output}} = \underbrace{\boxed{12}}_{\text{given input}}$$

algebraic equation

- The given input of the forward problem is unknown output of backward problem
- The unknown output of the foward problem is the given input of the backward problem

Let's look at the function

$$g(x) = 4 \cdot x$$

Forward Problem:

$$4 \cdot \boxed{5} = \boxed{y}$$

given input — unknown output

function evaluation

Backward Problem

$$4 \cdot \boxed{x} = \boxed{20}$$

unknown output — given input

algebraic equation

- The roles of x and y flip when we move from forward to backward problem.

Let's look at the function

$$m(x) = 2^{x} = y$$

Forward Problem:

given input

$$2^{\boxed{6}} = \boxed{y}$$

unknown output

Evaluate function

Backward Problem:

unknown output

$$2^{\boxed{x}} = \boxed{64}$$

given input

Algebraic equation

Think about this:

Forward Problem

given input

$$2^{\boxed{x}} = \boxed{y}$$

desired output

Function evaluation

inverse

Backward Problem

desired output

$$2^{\boxed{y}} = \boxed{x}$$

given input

algebraic equation

- In order to transform a forward problem to a backward problem we switch the $\boxed{y}$ and $\boxed{x}$

- to find an inverse function we switch the $\boxed{x}$ and $\boxed{y}$