

Math 48A, Lesson 12: Combining of Functions

1. COMPOSITION OF FUNCTIONS

COMPOSITION OF FUNCTIONS

Given two functions f and g , the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f . In other words, $(f \circ g)(x)$ is defined whenever both $g(x)$ and $f(g(x))$ are defined. We can picture $f \circ g$ using an arrow diagram (Figure 4).

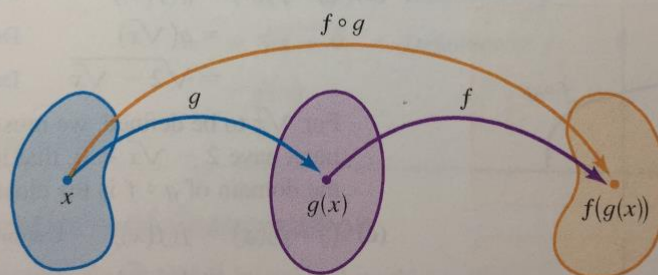


FIGURE 4 Arrow diagram for $f \circ g$

1A. Please translate this into abuelita language. In other words, translate this idea into language that your grandmother (abuelita) can understand.

1B. Consider the following functions

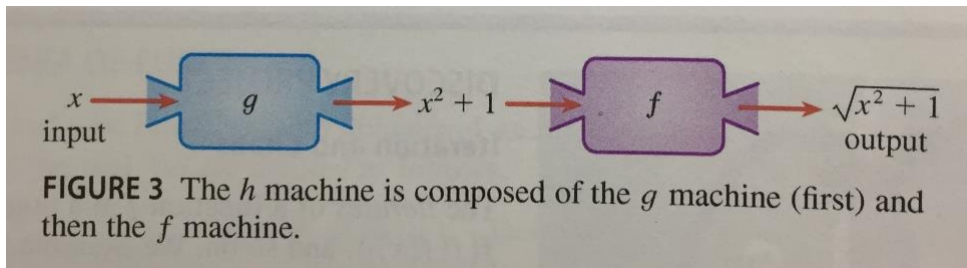
$$f(x) = \sqrt{x}$$

and

$$g(x) = x^2 + 1$$

We're going to study composition of functions in the form

$$f \circ g(x) = f(g(x))$$



2. COMPOSITIONS OF FUNCTIONS

Consider the following functions

$$f(x) = x^2 \quad \text{and} \quad g(x) = x + 3$$

Find each of the following functions:

2A. $f \circ g(x) = f(g(x))$

2B. $g \circ f(x) = g(f(x))$

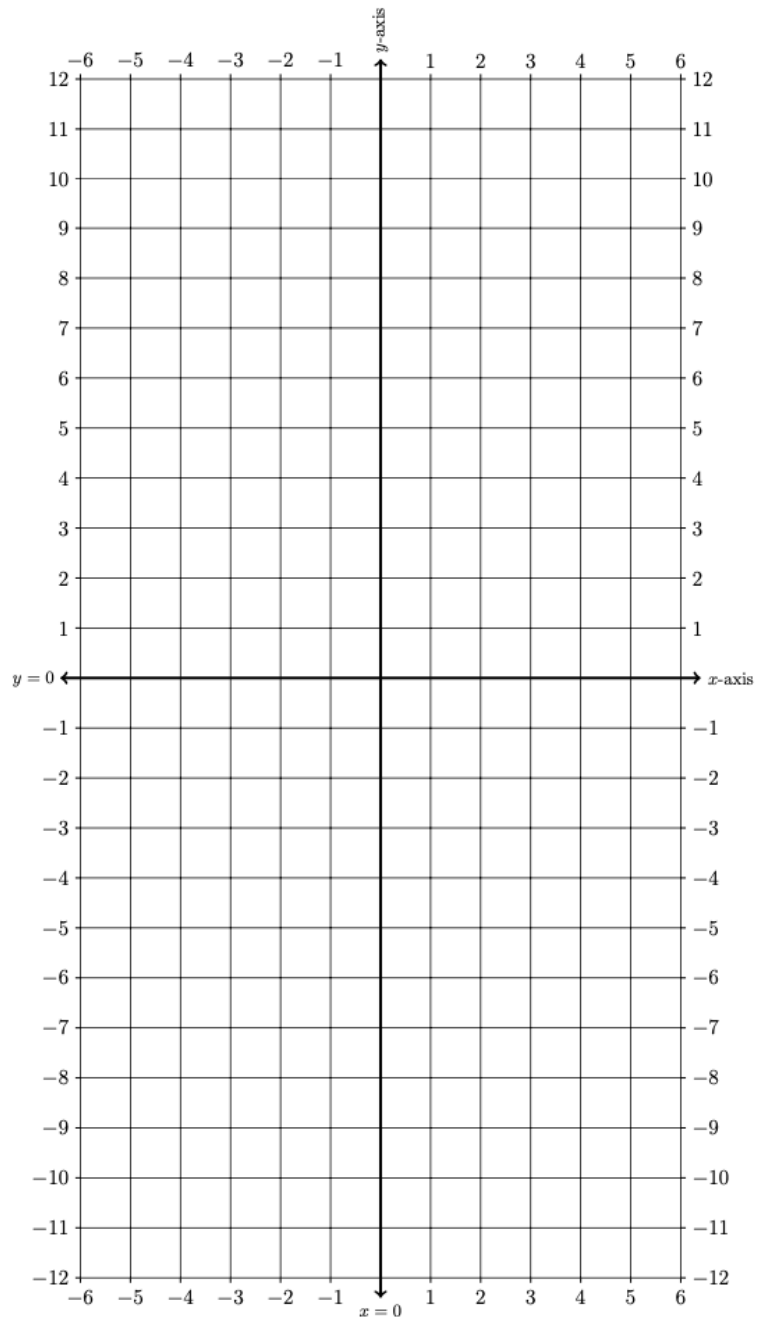
3. SUM AND DIFFERENCE OF FUNCTIONS

3A. Consider the following functions

$$f(x) = |x|, \quad g(x) = -x, \quad h(x) = (f + g)(x) = f(x) + g(x)$$

Create a table of values and graph the resulting parabolas on these axes below.

Input	Output		
	$f(x)$	$g(x)$	$h(x)$
x			
-6			
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			
5			
6			

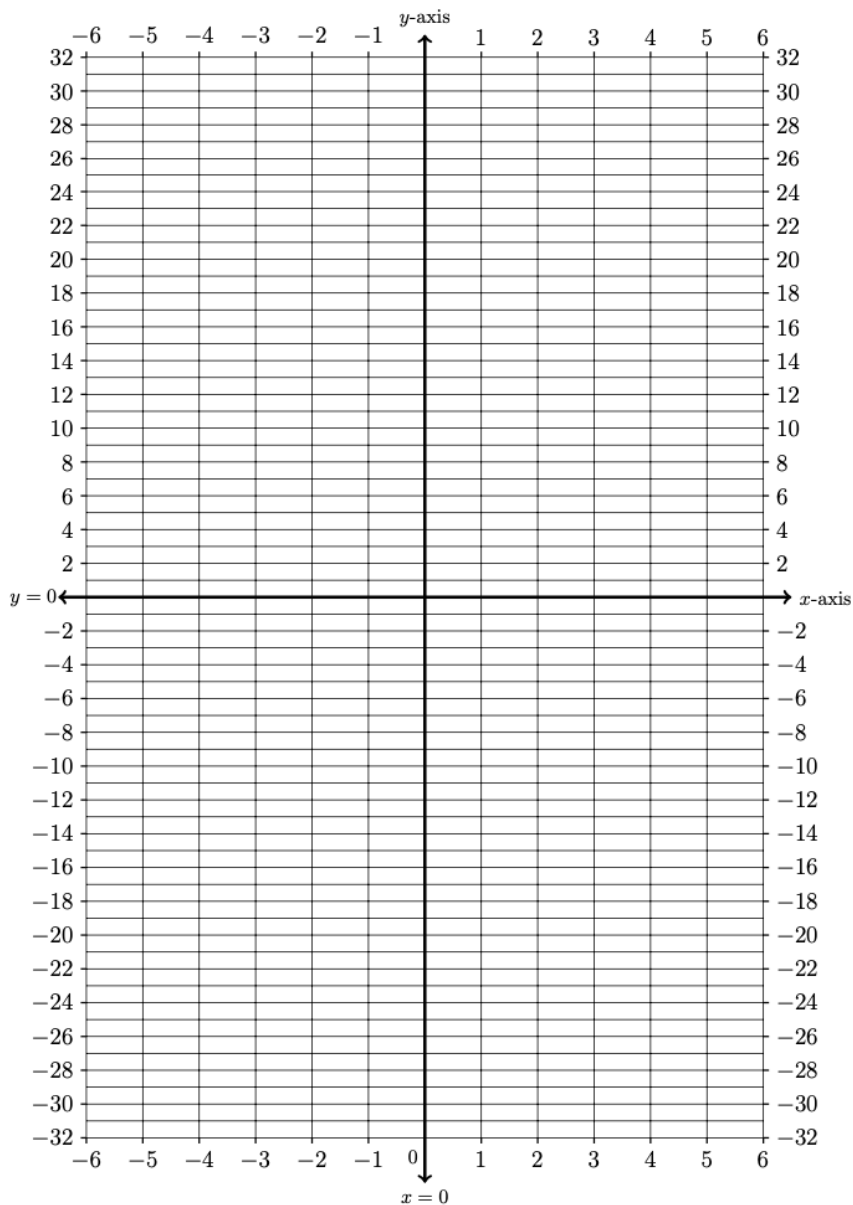


3B. Consider the following functions

$$f(x) = x^2, \quad g(x) = 3x + 4, \quad h(x) = (f - g)(x) = f(x) - g(x)$$

Create a table of values and graph the resulting parabolas on these axes below.

Input	Output			
	x	$f(x)$	$g(x)$	$h(x)$
-6				
-5				
-4				
-3				
-2				
-1				
0				
1				
2				
3				
4				
5				
6				



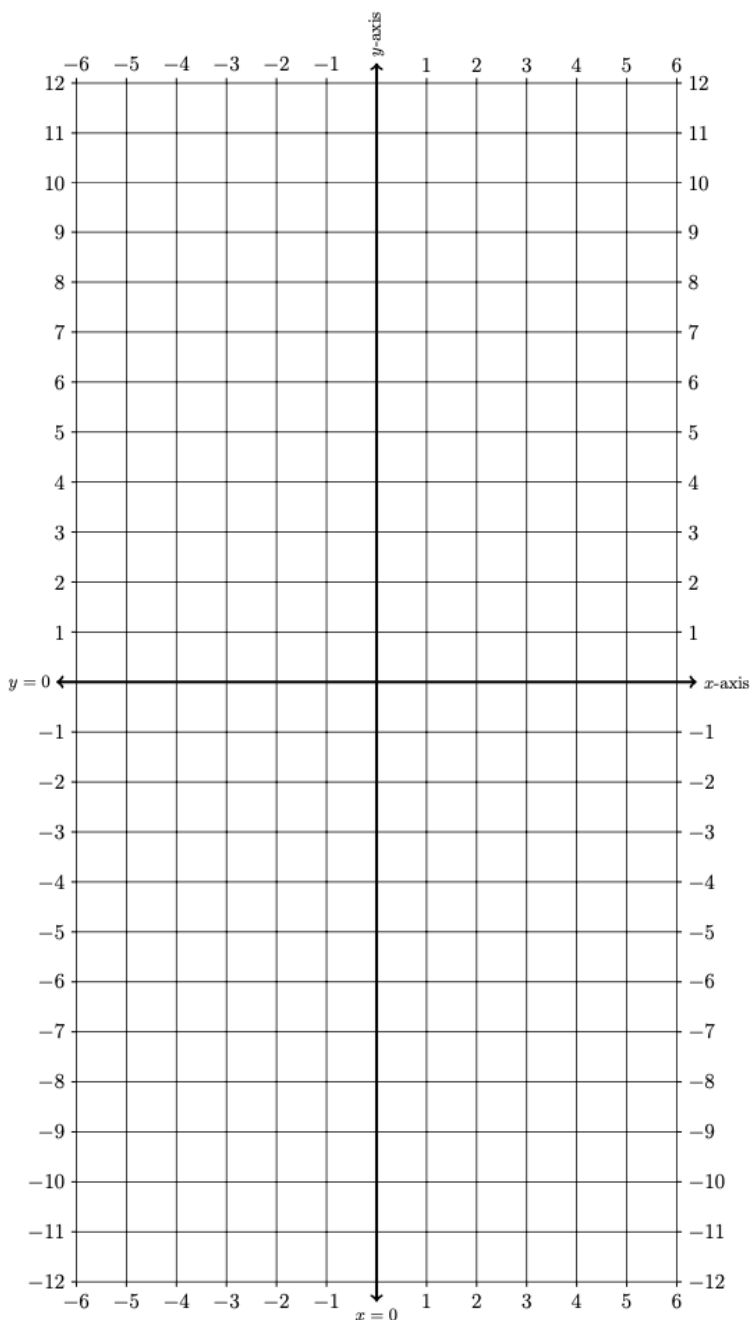
4. PRODUCT AND QUOTIENT OF FUNCTIONS

4A. Consider the following functions

$$f(x) = x^2, \quad g(x) = x^2 + 1, \quad h(x) = (f \div g)(x) = \frac{f(x)}{g(x)}$$

Create a table of values and graph the resulting curves on these axes below. Use Desmos.com to confirm your results

Input	Output		
x	$f(x)$	$g(x)$	$h(x)$
-6			
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			
5			
6			

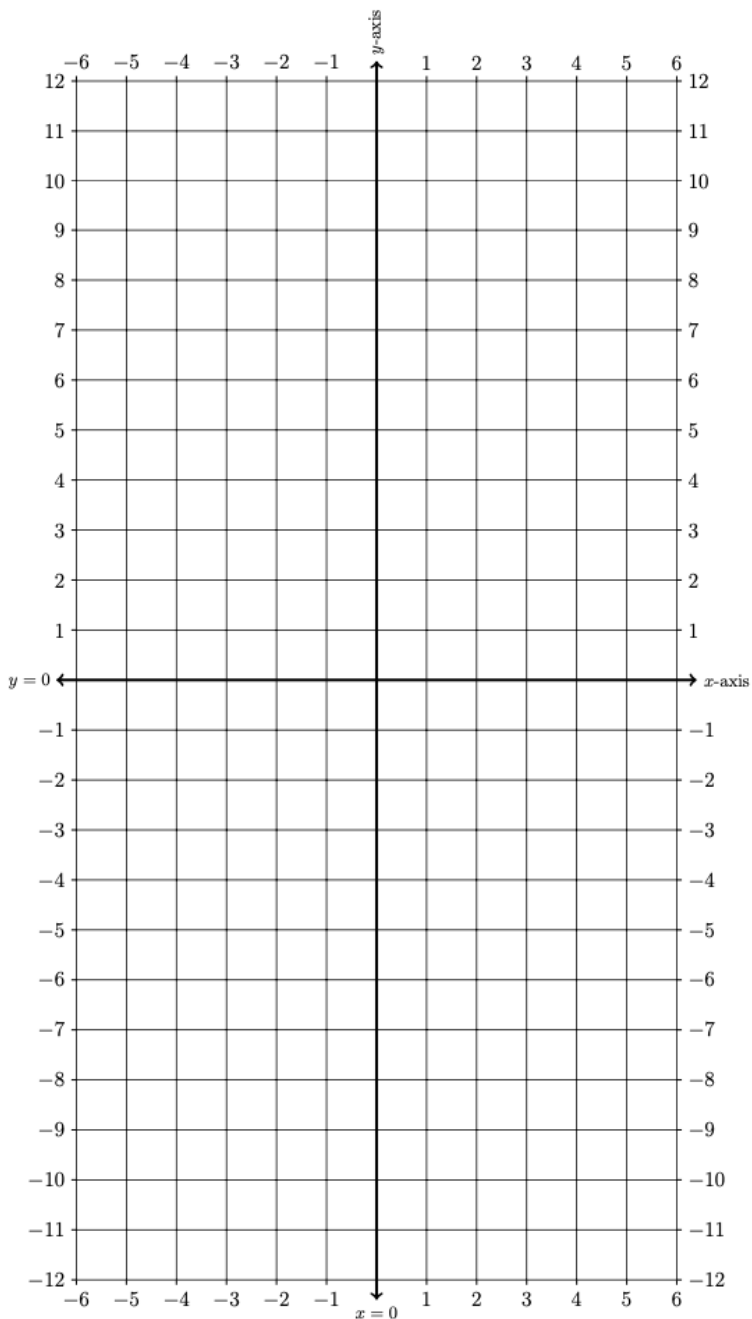


4B. Consider the following functions

$$f(x) = |x - 4|, \quad g(x) = x - 4, \quad h(x) = (f \div g)(x) = \frac{f(x)}{g(x)}$$

Create a table of values and graph the resulting curves on these axes below. Use Desmos.com to confirm your results

Input	Output			
	x	$f(x)$	$g(x)$	$h(x)$
-6				
-5				
-4				
-3				
-2				
-1				
0				
1				
2				
3				
4				
5				
6				

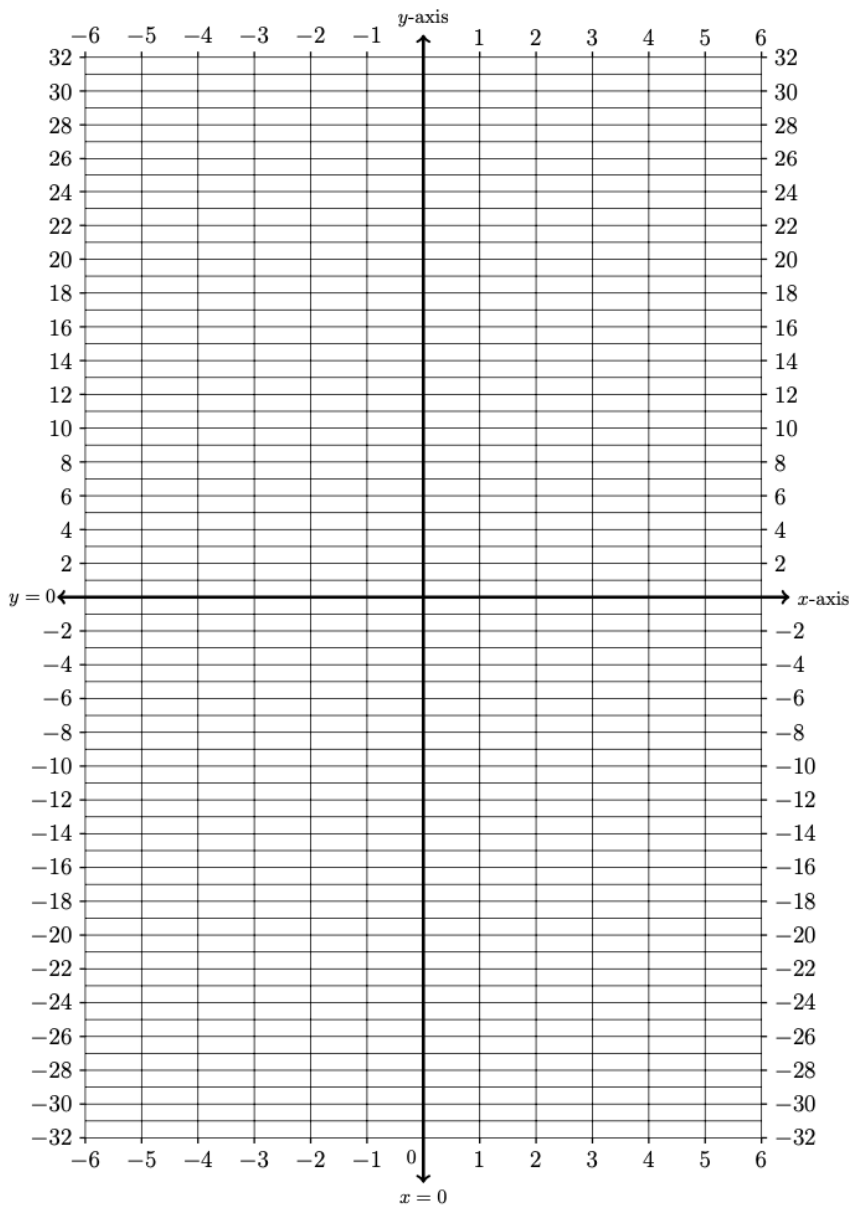


4C. Consider the following functions

$$f(x) = x^2 - 4, \quad g(x) = x, \quad h(x) = (f \cdot g)(x) = f(x) \cdot g(x)$$

Create a table of values and graph the resulting curves on these axes below. Use Desmos.com to confirm your results

Input	Output		
	$f(x)$	$g(x)$	$h(x)$
x			
-6			
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			
5			
6			



5. PRACTICE COMBINING FUNCTIONS

Consider the following functions

$$f(x) = x - 1$$

and

$$g(x) = x^2 - 1$$

Please find each of the following:

5A. $(f + g)(x)$

5B. $(f - g)(x)$

5C. $(f \cdot g)(x)$

5D. $(f \div g)(x)$

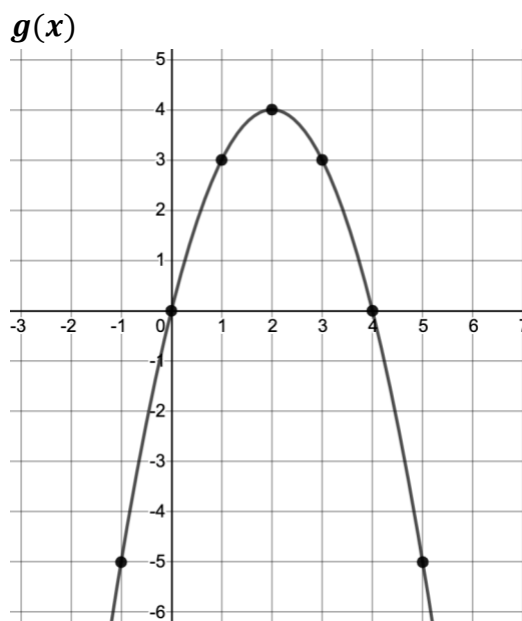
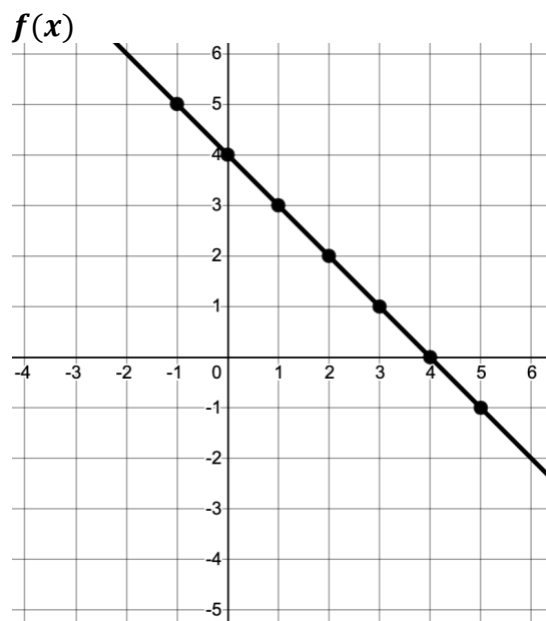
5E. $f \circ g(x) = f(g(x))$

5F. $g \circ f(x) = g(f(x))$

Show your work and simplify whenever you can.

6. MORE PRACTICE COMBINING FUNCTIONS

Below are graphs of functions $f(x)$ and $g(x)$.



Use the graphs above to evaluate each of the following.

6A. $f(g(1))$

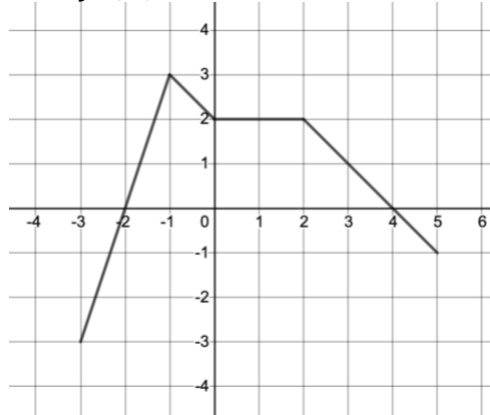
6B. $(g \circ f)(2)$

6C. $(f + g)(-1)$

6D. $\left(\frac{f}{g}\right)(0)$

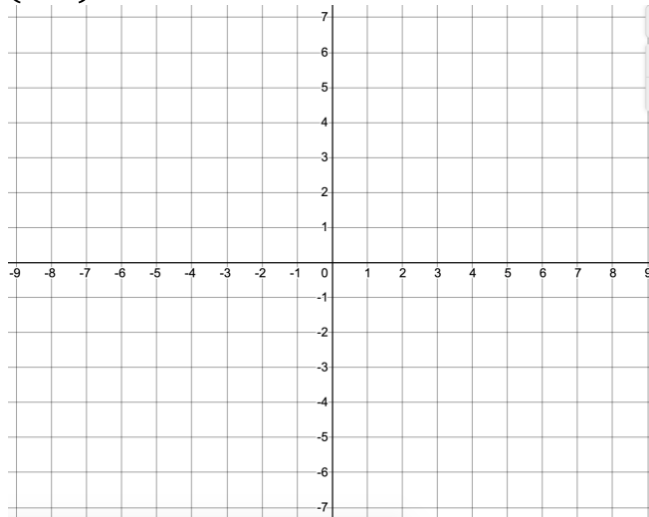
7. PRACTICE WITH TRANSFORMATIONS

Below is a graph of a function $f(x)$



Using the work you did on Lessons 10 and 11, complete each of the following problems. Make your best educated guesses using your brain first. Then confirm your guesses using Desmos.com

7A. Sketch a graph of $2f(-x)$.



7B. Sketch a graph of $3 - f(x + 2)$.

