Name:
Math 48A, Lesson 11: Transformations of Functions (Part 2)

## 1. COMBINE HORIZONTAL AND VERTICAL SHIFTS

## 1A. Consider the following quadratic functions

$$
f(x)=x^{2}, \quad g(x)=f(x-3)+4, \quad h(x)=f(x+2)-12
$$

Create a table of values and graph the resulting parabolas on these axes below.

| Input | Output |  |  |
| :---: | :---: | :---: | :---: |
| $x$ | $f(x)$ | $g(x)$ | $h(x)$ |
| -6 |  |  |  |
| -5 |  |  |  |
| -4 |  |  |  |
| -3 |  |  |  |
| -2 |  |  |  |
| -1 |  |  |  |
| 0 |  |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |



1B. Make a conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f(x)$ and constants $h, k>0$. Suppose we define functions

$$
g(x)=f(x-h)+k
$$

What is the relationship between $f(x)$ and $g(x)$ ? What happens if constant $h$ is positive or negative? What happens if constant $k$ is positive or negative?

## 2. REFLECTING GRAPHS ABOUT THE X-AXIS

2A. Let's consider how to "reflect" the graph of a function about the $x$-axis. To do so, consider the following functions

$$
f(x)=|x| \quad \text { and } \quad g(x)=-f(x)
$$

Create a table of values and graph the resulting curves on these axes below.

| Input | Output |  |
| :---: | :---: | :---: |
| $x$ | $f(x)$ | $g(x)$ |
| -6 |  |  |
| -5 |  |  |
| -4 |  |  |
| -3 |  |  |
| -2 |  |  |
| -1 |  |  |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |



2B. Let's consider how to "reflect" the graph of a function about the $x$-axis. To do so, consider the following functions

$$
f(x)=x^{2} \quad \text { and } \quad g(x)=-f(x)
$$

Create a table of values and graph the resulting curves on these axes below.

| Input | Output |  |
| :---: | :---: | :---: |
| $x$ | $f(x)$ | $g(x)$ |
| -6 |  |  |
| -5 |  |  |
| -4 |  |  |
| -3 |  |  |
| -2 |  |  |
| -1 |  |  |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |



2C. Suppose we have a function $f(x)$ and we define a new function $g(x)=-f(x)$. Based on your work in Problems 2A and 2B, make a conjecture about the relationship between the graphs of $f(x)$ and $g(x)$. Explain why you think your conjecture might be true.
$\qquad$

## 3. REFLECTION ABOUT VERTICAL AXIS

Consider the following shifts of the input variable


Draw the effect of this shift on the real number line ( $x$-axis) below:

Shifted values:
$x$-values:


What do you notice about the reflexed input $-x$ versus the original input $x$ ?

## 4. REFLECTING GRAPHS ABOUT THE Y-AXIS

4A. Let's consider how to "reflect" the graph of a function about the $x$-axis. To do so, consider the following functions

$$
f(x)=\sqrt{x} \quad \text { and } \quad g(x)=f(-x)
$$

Create a table of values and graph the resulting curves using Desmos.com

4B. Let's consider how to "reflect" the graph of a function about the $x$-axis. To do so, consider the following functions

$$
f(x)=x^{3} \quad \text { and } \quad g(x)=f(-x)
$$

Create a table of values and graph the resulting curves using Desmos.com

4C. Suppose we have a function $f(x)$ and we define a new function $g(x)=f(-x)$. Based on your work in Problems 4A and 4B, make a conjecture about the relationship between the graphs of $f(x)$ and $g(x)$. Explain why you think your conjecture might be true.

