### Math 48A, Lesson 11: Transformations of Functions (Part 2)

### 1. COMBINE HORIZONTAL AND VERTICAL SHIFTS

1A. Consider the following quadratic functions

$$f(x) = x^2$$
,  $g(x) = f(x - 3) + 4$ ,  $h(x) = f(x + 2) - 12$ 

Create a table of values and graph the resulting parabolas on these axes below.

Input			$-6$ -5 -4 -3 -2 -1 $\xrightarrow{y-\text{axis}}$ 1 2 3											5	6		
		Output		32 - 30 -										-			+ 32 - 30
x	f(x)	g(x)	h(x)	28 -													-30 - 28
				26 -													26
-6				20				-						_	-	-	24
				22 -													22
				20 -				-							-	-	20
-5				18 -				-							-	-	- 18
				16 -				-							-	-	16
				14 -				-							-		- 14
-4				12 -													12
				10 -													10
				8 -													8
-3				6 -													6
				4 -													4
				2 -													2
-2				$y = 0 \longleftarrow$													$\rightarrow x - i$
				-2 -													-2
				-4 -				-							-	-	-4
				-6 -				-							-	-	-6
-1				-8 -											-		8
				-10 -				-							-	-	-10
0				-12 -				-	-					-	-	-	-12
				-14 -				-							-	-	-14
				-16 -				-							-	-	-16
				-18 - 20 - 20 - 20 - 20 - 20 - 20 - 20 - 2										_		-	-18 -20
1				-20 - -22 -													-20 -22
				-22 -													-22 -24
				-24 - 26 -													-24 -26
2				-20 - 28 - 28 - 28 - 28 - 28 - 28 - 28 -													-20 -28
				-20													-20 - 30
				-30													-30
3				-02	-6 -1	5 –	4 -	-3 -	-2 -	1 0	1	2	2	3	4	5	6
										x =	= 0						
4																	
				1													
5																	
				1													
6																	

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1B. Make a conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function f(x) and constants h, k > 0. Suppose we define functions

$$g(x) = f(x - h) + k$$

What is the relationship between f(x) and g(x)? What happens if constant h is positive or negative? What happens if constant k is positive or negative?

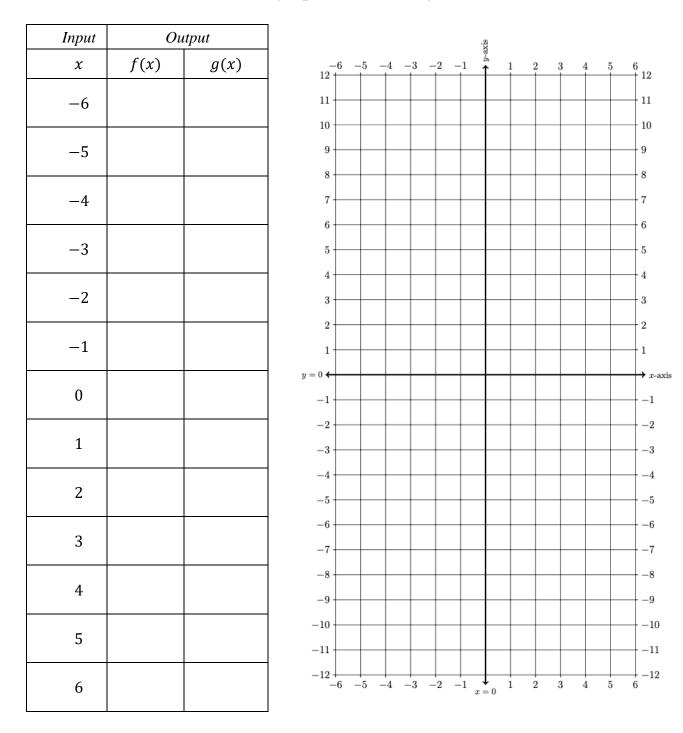
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### 2. REFLECTING GRAPHS ABOUT THE X-AXIS

2A. Let's consider how to "reflect" the graph of a function about the x –axis. To do so, consider the following functions

$$f(x) = |x|$$
 and  $g(x) = -f(x)$ .

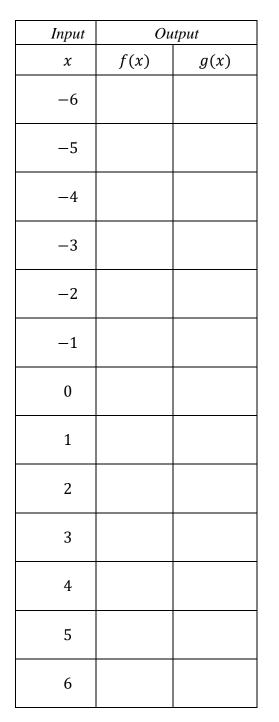
Create a table of values and graph the resulting curves on these axes below.

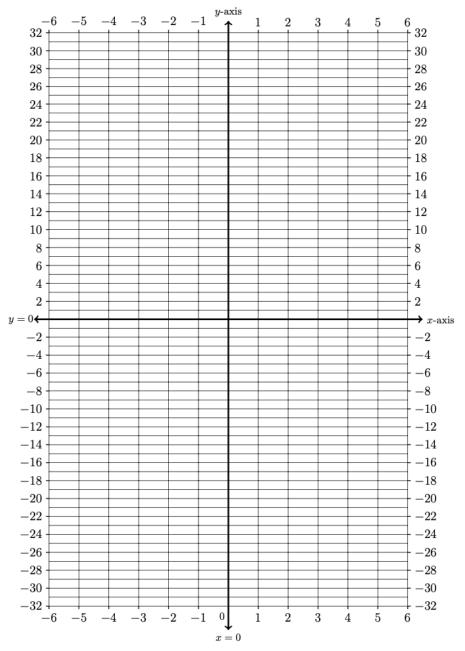


2B. Let's consider how to "reflect" the graph of a function about the x –axis. To do so, consider the following functions

$$f(x) = x^2$$
 and  $g(x) = -f(x)$ .

Create a table of values and graph the resulting curves on these axes below.



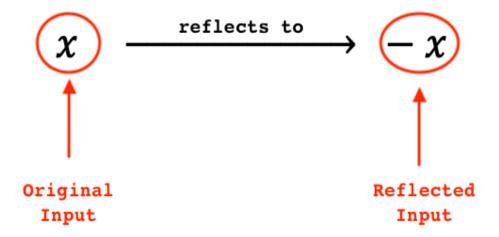


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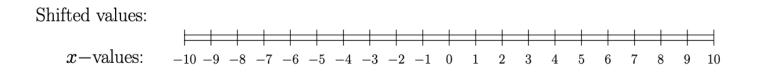
2C. Suppose we have a function f(x) and we define a new function g(x) = -f(x). Based on your work in Problems 2A and 2B, make a conjecture about the relationship between the graphs of f(x) and g(x). Explain why you think your conjecture might be true.

## 3. REFLECTION ABOUT VERTICAL AXIS

Consider the following shifts of the input variable



Draw the effect of this shift on the real number line (x - axis) below:



What do you notice about the reflexed input -x versus the original input x?

# 4. REFLECTING GRAPHS ABOUT THE Y-AXIS

4A. Let's consider how to "reflect" the graph of a function about the x –axis. To do so, consider the following functions

$$f(x) = \sqrt{x}$$
 and  $g(x) = f(-x)$ .

Create a table of values and graph the resulting curves using Desmos.com

4B. Let's consider how to "reflect" the graph of a function about the x –axis. To do so, consider the following functions

$$f(x) = x^3$$
 and  $g(x) = f(-x)$ .

Create a table of values and graph the resulting curves using Desmos.com

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4C. Suppose we have a function f(x) and we define a new function g(x) = f(-x). Based on your work in Problems 4A and 4B, make a conjecture about the relationship between the graphs of f(x) and g(x). Explain why you think your conjecture might be true.