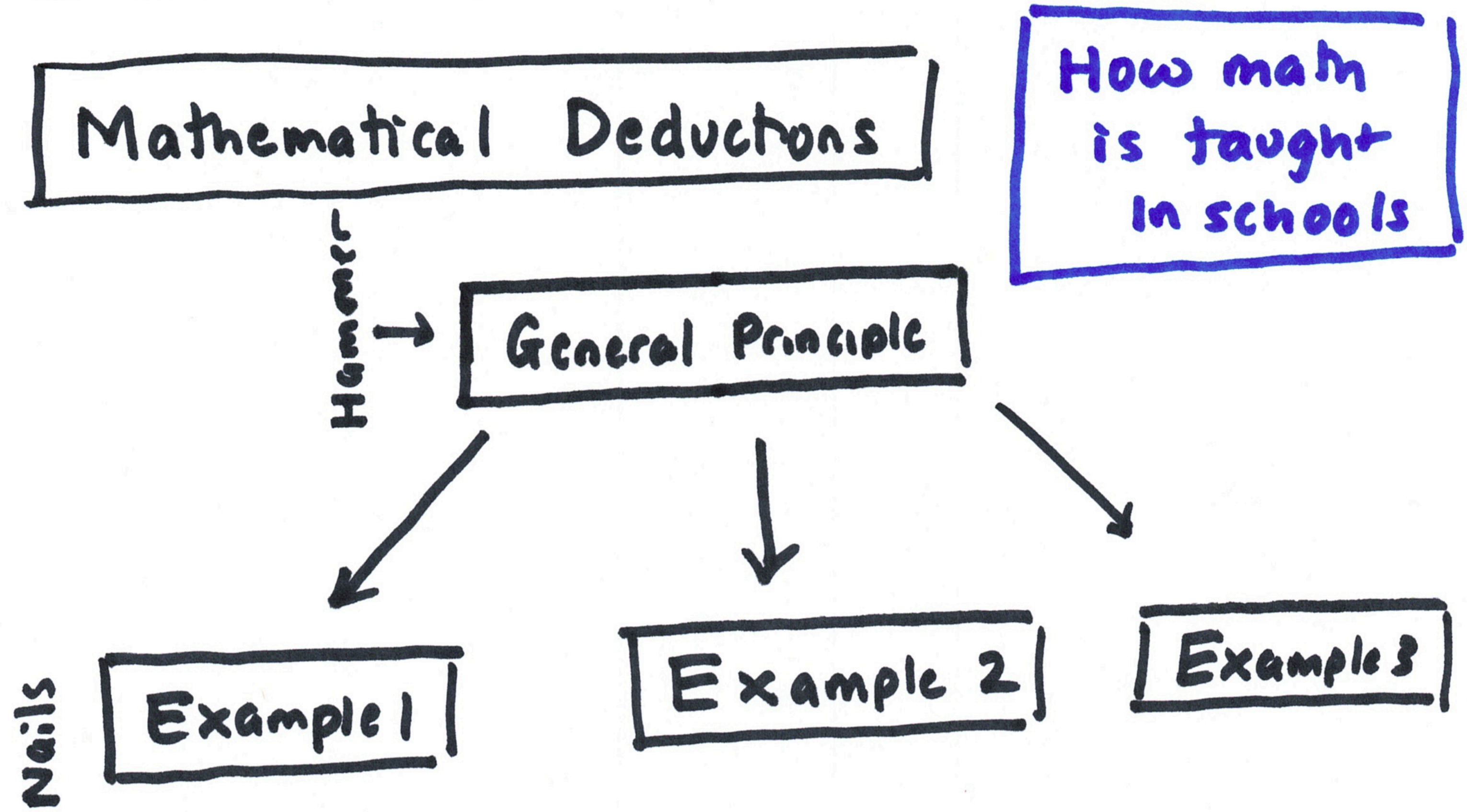
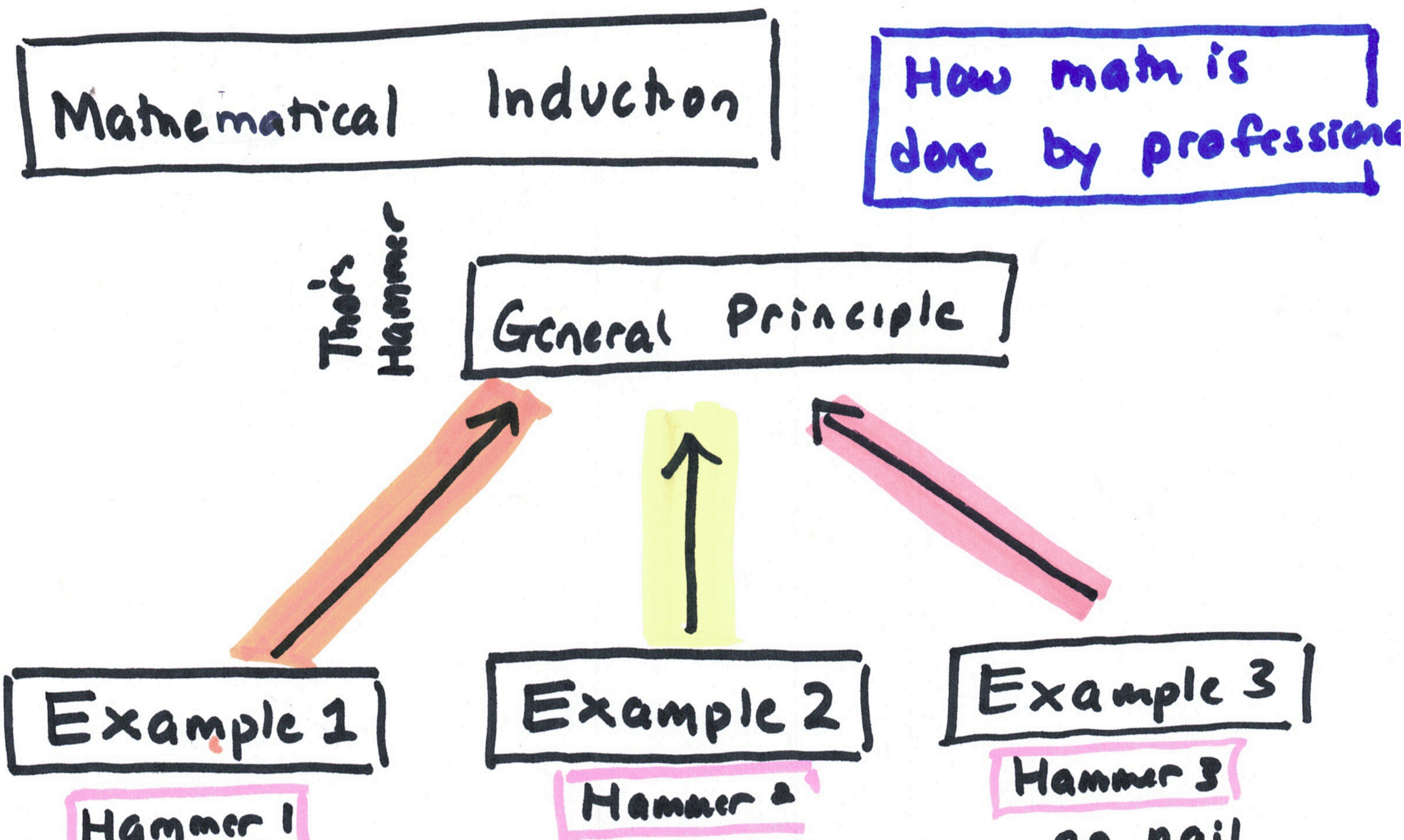
Lesson 10 Solutions, part 1



Name:________________________ Class #:______

# Math 48A, Lesson 10: Transformations of Functions (Part 1)

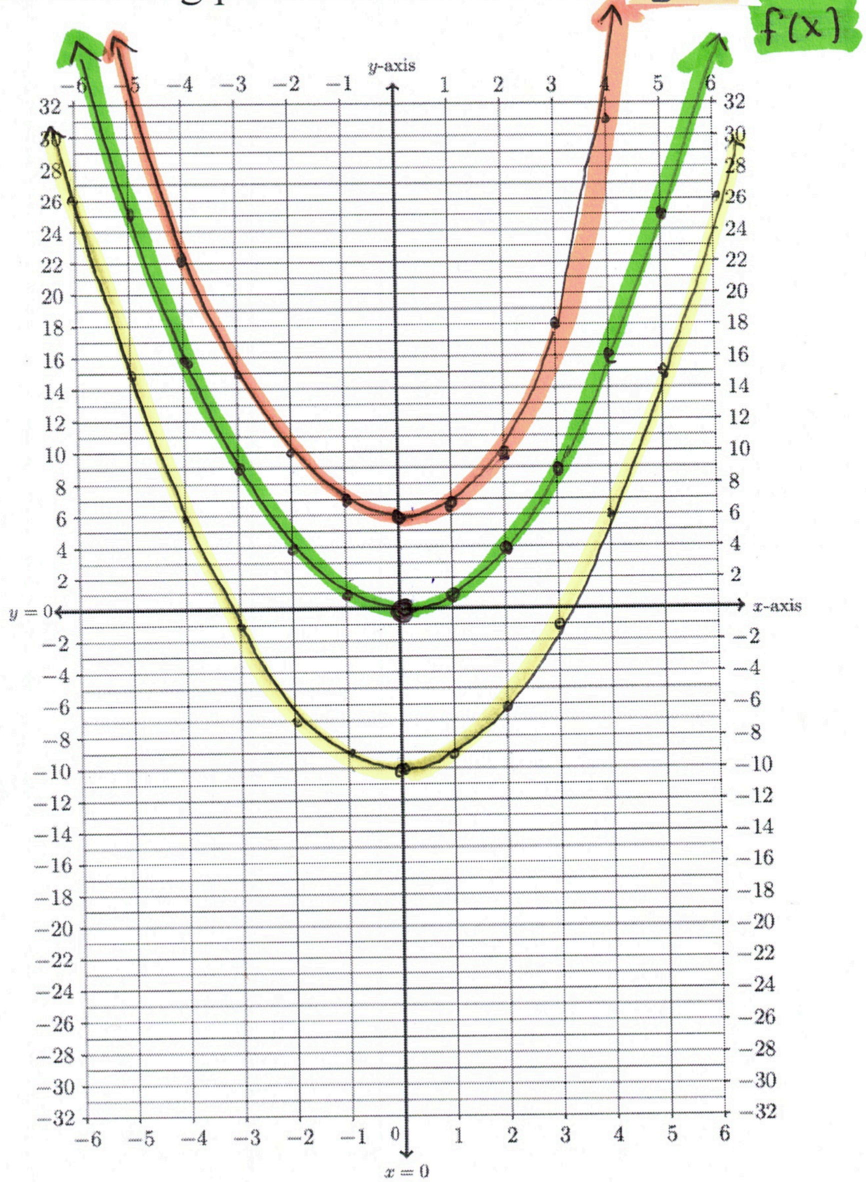
## 1. VERTICAL SHIFTS OF A QUADRATIC FUNCTION

1A. Consider the following quadratic functions

$$\boxed{f(x) = x^2}, \quad \boxed{g(x) = f(x) + 6}, \quad \boxed{h(x) = f(x) - 10}$$

Create a table of values and graph the resulting parabolas on these axes

| *Input* | *Output* | | |
|---|---|---|---|
| $x$ | $f(x)$ | $g(x)$ | $h(x)$ |
| −6 | 36 | 42 | 26 |
| −5 | 25 | 31 | 15 |
| −4 | 16 | 22 | 6 |
| −3 | 9 | 15 | -1 |
| −2 | 4 | 10 | -6 |
| −1 | 1 | 7 | -9 |
| 0 | 0 | 6 | -10 |
| 1 | 1 | 7 | -9 |
| 2 | 4 | 10 | -6 |
| 3 | 9 | 15 | -1 |
| 4 | 16 | 22 | 6 |
| 5 | 25 | 31 | 15 |
| 6 | 36 | 42 | 26 |




parabola's are symmetric around the vertex

$$g(x) = \boxed{f(x)} + 6$$
$$\Rightarrow g(x) = \boxed{\boxed{x^2} + 6}$$

to get the orange value from green values, we add 6

$$h(x) = \boxed{f(x)} - 10$$
$$= \boxed{\boxed{x^2} - 10}$$

1

Problem 1B

Notice:

for $\boxed{g(x)} = \boxed{f(x)} + 6$

- ☐ the "starting" point (vertex) of the orange parabola is 6 units above the vertex of the green parabola

  (vertex is the point in the center of parabola at the bottom)

for $\boxed{h(x)} = \boxed{f(x)} - 10$

- ☐ ¿ the share the same slope?
- ☐ they all share $x^2$
- ☐ their all symmetric about their minimum value
- ☐ take the vertex of green and subtract 10 to get yellow (shift the graph down by 10)

Name:______________________________ Class #:______

1C. Make a conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f(x)$ and a positive constant $c > 0$. Suppose we define functions

$$g(x) = \boxed{\boxed{f(x)} + c} \qquad \text{and} \qquad h(x) = f(x) - c$$

What is the relationship between $f(x)$, $g(x)$, and $h(x)$?

when we take

$\boxed{g(x)} = \boxed{f(x)} + C$

we do the following:

- ☐ we shift the graph of $\boxed{f(x)}$ upwards by c amaunt of units to get the graph of $\boxed{g(X)}$

- ☐ we shift the green graph vertically by c units to get the orange graph

③

ut

Name: Homework Pass the Torch STEM Center Class #:

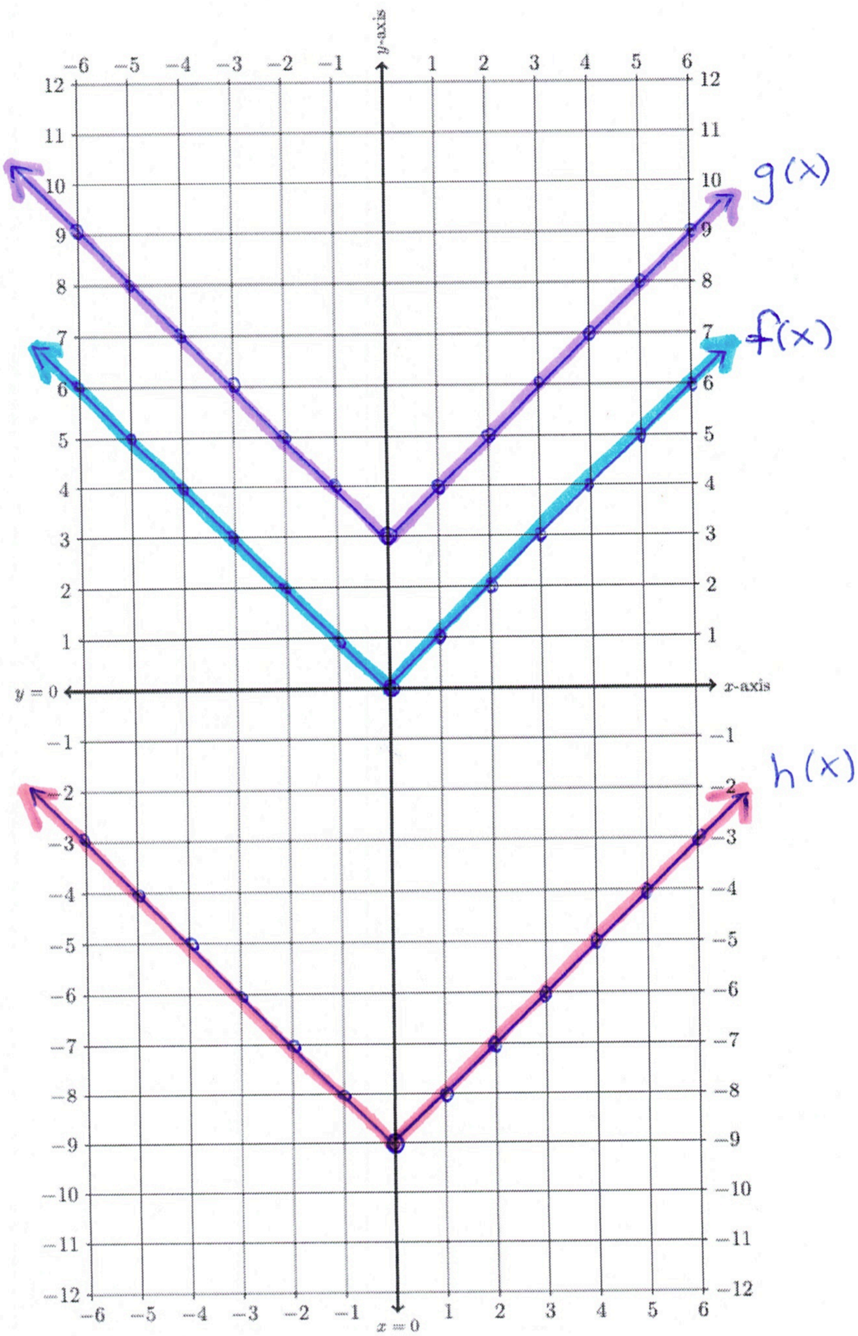
## 2. VERTICAL SHIFTS OF AN ABSOLUTE VALUE FUNCTION

2A. Let's test your conjecture from Problem 1C on a different type of function. Consider the following absolute value functions

$\boxed{f(x) = |x|,}$ $\boxed{g(x) = f(x) + 3,}$ $\boxed{h(x) = f(x) - 9}$

For each function, specifically identify the value of positive constant $c > 0$. Then, create a table of values and graph the resulting curves on these axes below

| *Input* | *Output* | | |
|---|---|---|---|
| $x$ | $f(x)$ | $g(x)$ | $h(x)$ |
| $-6$ | 6 | 9 | -3 |
| $-5$ | 5 | 8 | -4 |
| $-4$ | 4 | 7 | -5 |
| $-3$ | 3 | 6 | -6 |
| $-2$ | 2 | 5 | -7 |
| $-1$ | 1 | 4 | -8 |
| 0 | 0 | 3 | -9 |
| 1 | 1 | 4 | -8 |
| 2 | 2 | 5 | -7 |
| 3 | 3 | 6 | -6 |
| 4 | 4 | 7 | -5 |
| 5 | 5 | 8 | -4 |
| 6 | 6 | 9 | -3 |



Name:______________________________ Class #:______

2B. Look back at both the graphs and the table of values from Problem 2A. What do you notice about the relationship between the output values of the functions

$f(x)$, $g(x) = \boxed{f(x) + 3}$, $h(x) = \boxed{f(x) - 9}$

Notice for $\boxed{g(x)} = \boxed{f(x) + 3}$

- ☐ the "starting" point (vertex) of the purple absolute value curve is three units above the blue absolute value curve
- ☐ the purple graph of function $g(x)$ is shifted three units above the blue graph of function $f(x)$

For $h(x) = \boxed{f(x) - 9}$

- ☐ the starting point (vertex) of the pink absolute value curve is nine units below the blue absolute value curve.
- ☐ the pink graph of function $h(x)$ is shifted nine units below the blue graph of function $f(x)$

5

Name:_______________ Class #:_____

2C. Revise and update your conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f(x)$ and a positive constant $c > 0$.
Suppose we define functions

$\boxed{\text{I.}}$ $g(x) = \boxed{f(x) + c}$ and $\boxed{\text{II.}}$ $h(x) = \boxed{f(x) - c}$

What is the relationship between $f(x)$, $g(x)$, and $h(x)$? Try to put this in both nerdy mathematical language and abuelita language

$\boxed{\text{I.}}$ When we take $c > 0$ and form $g(x) = f(x) + c$

- ☐ we shift the graph of $f(x)$ upwards by $c$ units to get the graph of $g(x)$
- ☐ this is equivalent to moving all output values of $f(x)$ vertically in the positive direction (upwards) by $c$ units (also known as a vertical shift).

$\boxed{\text{II.}}$ When we take $c > 0$ and form $h(x) = f(x) - c$

- ☐ we shift the graph of $f(x)$ down by $c$ units to get the graph of $h(x)$
- ☐ this is the same as subtracting from each output value of $f(x)$ the number $c$ which moves these points downwards by $c$ units and is known as a vertical shift in negative direction.

Name:________________________________ Class #:______

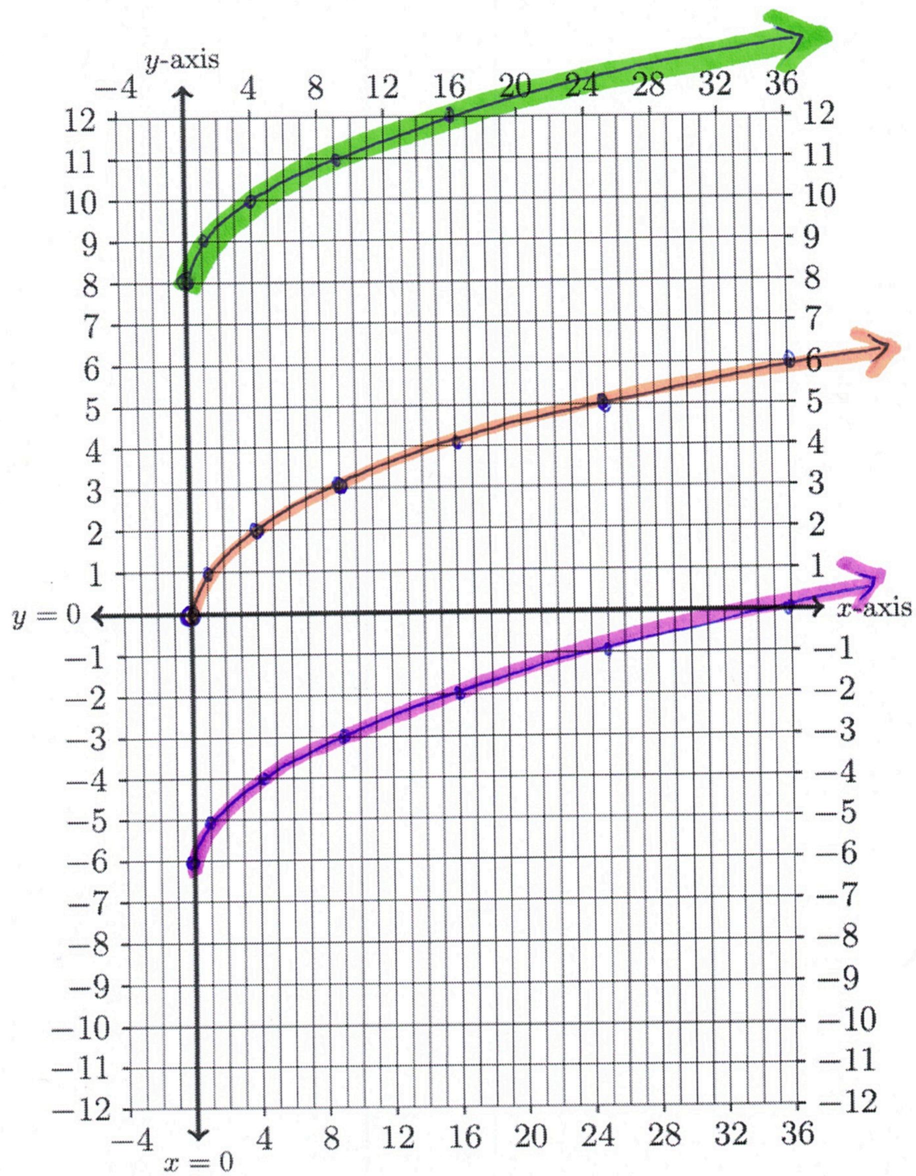
## 3. VERTICAL SHIFTS OF A ROOT FUNCTION

3A. Let's test the second draft of your conjecture from Problem 2C on a different type of function. Consider the following absolute value functions

$f(x) = \sqrt{x}$ , $\quad g(x) = f(x) + 8,$ $\quad h(x) = f(x) - 6$

For each function, specifically identify the value of positive constant $c > 0$. Then, create a table of values for $f(x)$. Without creating a table for $g(x)$ or $h(x)$, see if you can graph the resulting root function on the axes below.

| *Input* | *Output* |
|---|---|
| $x$ | $f(x)$ |
| $-1$ | undefined |
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |
| 9 | 3 |
| 16 | 4 |
| 25 | 5 |
| 36 | 6 |




Recall: if $x = -1$, then $f(x) = f(-1) = y = \sqrt{-1}$

$\Leftrightarrow \quad y^2 = -1$ ← NOT possible

Problem 3B)

Below we confirm our work from problem 3A using Desmos.com.

Untitled Graph desmos Log In or Sign Up

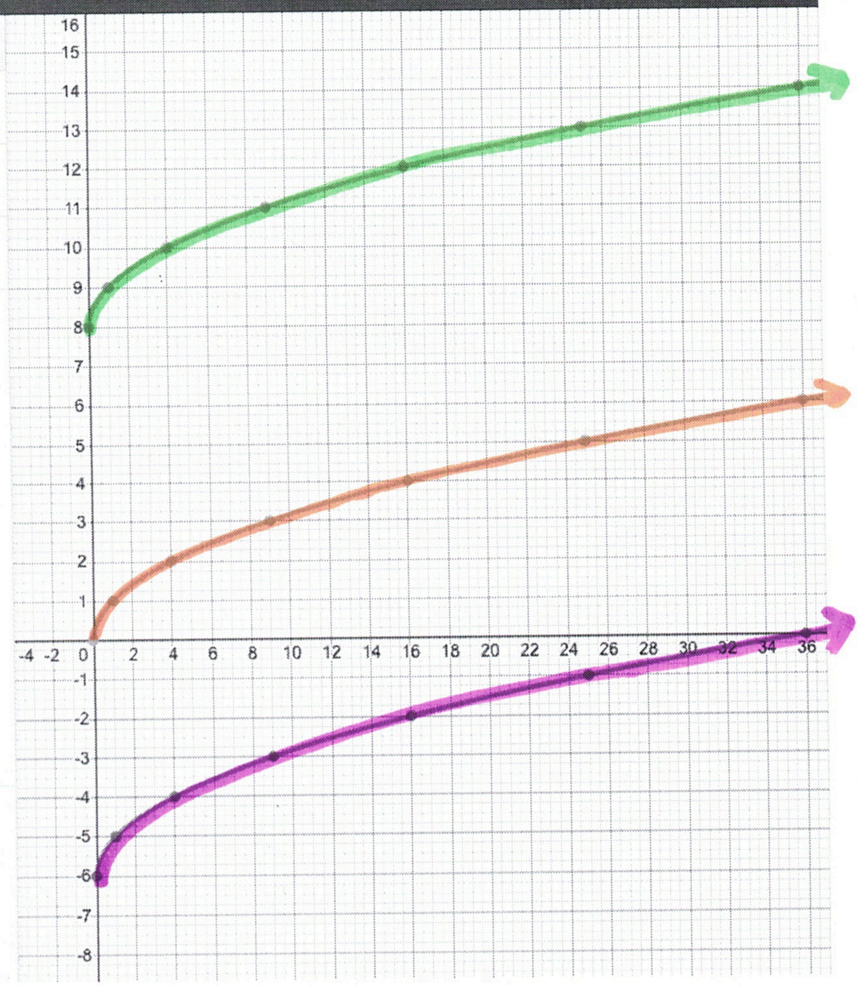
Open Graph (cmd+o)

1 $f(x) = \sqrt{x}$

2 $g(x) = f(x) + 8$

3 $h(x) = f(x) - 6$

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ |
|---|---|---|---|
| −1 | undefined | undefined | undefined |
| 0 | 0 | 8 | −6 |
| 1 | 1 | 9 | −5 |
| 4 | 2 | 10 | −4 |
| 9 | 3 | 11 | −3 |
| 16 | 4 | 12 | −2 |
| 25 | 5 | 13 | −1 |
| 36 | 6 | 14 | 0 |
| 49 | 7 | 15 | 1 |



Name:______________________ Class #:______

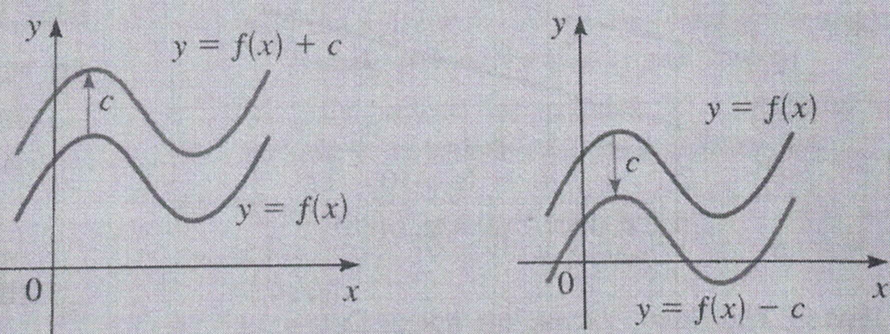
3C. Take a look at the call out box below:

**VERTICAL SHIFTS OF GRAPHS**

Suppose $c > 0$.

To graph $y = f(x) + c$, shift the graph of $y = f(x)$ upward $c$ units.

To graph $y = f(x) - c$, shift the graph of $y = f(x)$ downward $c$ units.




Translate this into abuelita language for yourself (simple non-technical language that describes what the math is saying so that your abuelita can understand). Really push yourself to make the description simple.

If we let $c > 0$ so that the letter c is a positive number, then

- ☐ for $g(x) = f(x) + C$ we notice our addition happens outside the parenthesis which effects the output values.
- ☐ To create the graph of g(x), we grab the entire graph of f(x) and drag it upwards by c units.

□ for $h(x) = f(x) - c$ we notice the subtraction happens outside the parenthesis which effects the output values

□ To create the graph of $h(x)$, we pull the graph of $f(x)$ straight downwards by c units.

Then notice the features

upward movement

$$g(x) = f(x) + c$$

Shift outside parenthesis
effects the output

$$h(x) = f(x) - c$$

downward
movement