Name:_____ Class #:_____

Math 48A, Lesson 10: Transformations of Functions (Part 1)

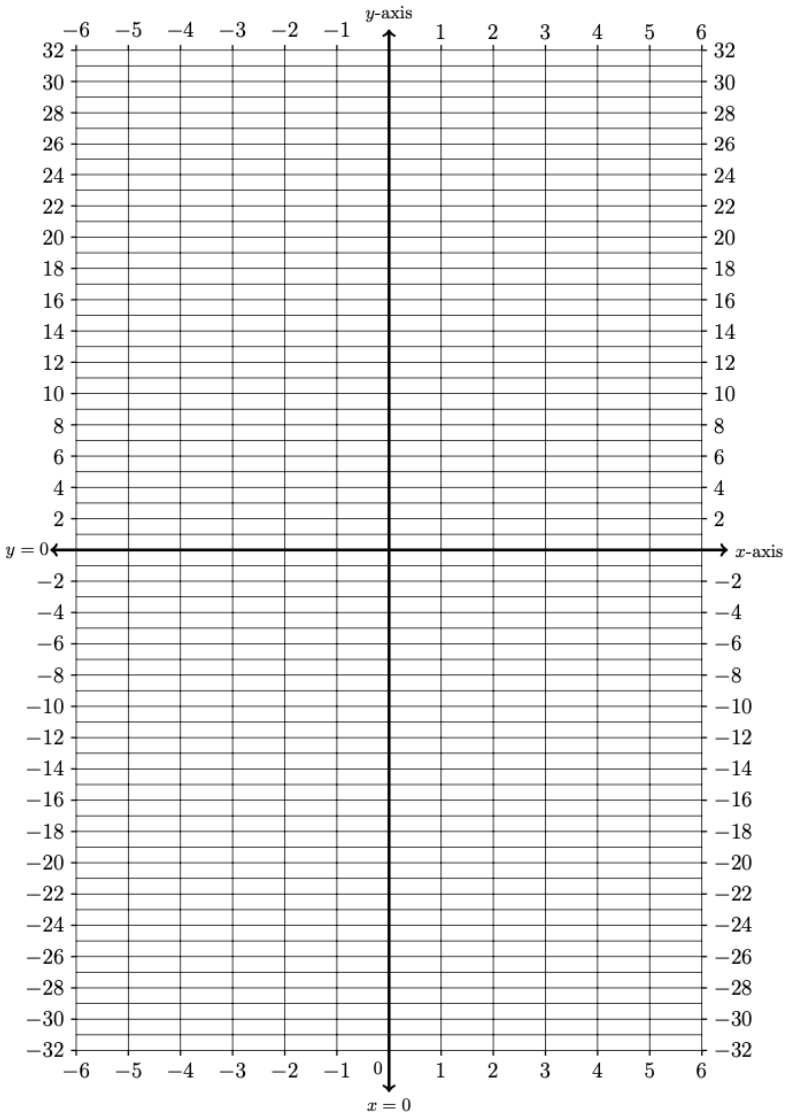
1. VERTICAL SHIFTS OF A QUADRATIC FUNCTION

1A. Consider the following quadratic functions

$$f(x) = x^2, \qquad g(x) = f(x) + 6, \qquad h(x) = f(x) - 10$$

Create a table of values and graph the resulting parabolas on these axes below.

Input	*Output*		
x	$f(x)$	$g(x)$	$h(x)$
-6			
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			
5			
6			



1B. Look back at both the graphs and the table of values from Problem 1A. What do you notice about the relationship between the output values of the functions

$$f(x), \qquad g(x) = f(x) + 6, \quad \text{and} \qquad h(x) = f(x) - 10$$

1C. Make a conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f(x)$ and a positive constant $c > 0$. Suppose we define functions

$$g(x) = f(x) + c \qquad \text{and} \qquad h(x) = f(x) - c$$

What is the relationship between $f(x)$, $g(x)$, and $h(x)$?

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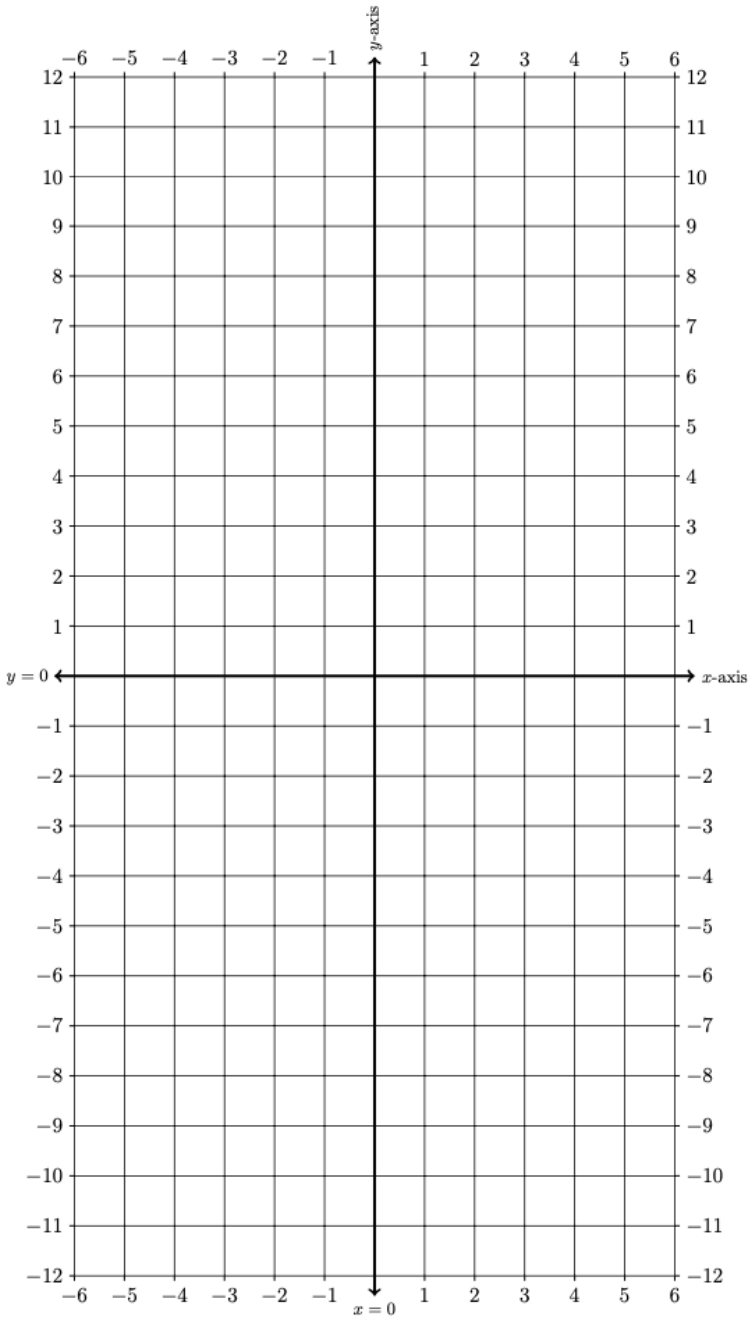
2. VERTICAL SHIFTS OF AN ABSOLUTE VALUE FUNCTION

2A. Let's test your conjecture from Problem 1C on a different type of function. Consider the following absolute value functions

$f(x) = |x|,$ $g(x) = f(x) + 3,$ $h(x) = f(x) - 9$

For each function, specifically identify the value of positive constant $c > 0$. Then, create a table of values and graph the resulting curves on these axes below

Input	*Output*		
x	$f(x)$	$g(x)$	$h(x)$
-6			
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			
5			
6			



2B. Look back at both the graphs and the table of values from Problem 2A. What do you notice about the relationship between the output values of the functions

$f(x)$, $g(x) = f(x) + 3$, $h(x) = f(x) - 10$

2C. Revise and update your conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f(x)$ and a positive constant $c > 0$.
Suppose we define functions

$$g(x) = f(x) + c \qquad \text{and} \qquad h(x) = f(x) - c$$

What is the relationship between $f(x)$, $g(x)$, and $h(x)$? Try to put this in both nerdy mathematical language and abuelita language

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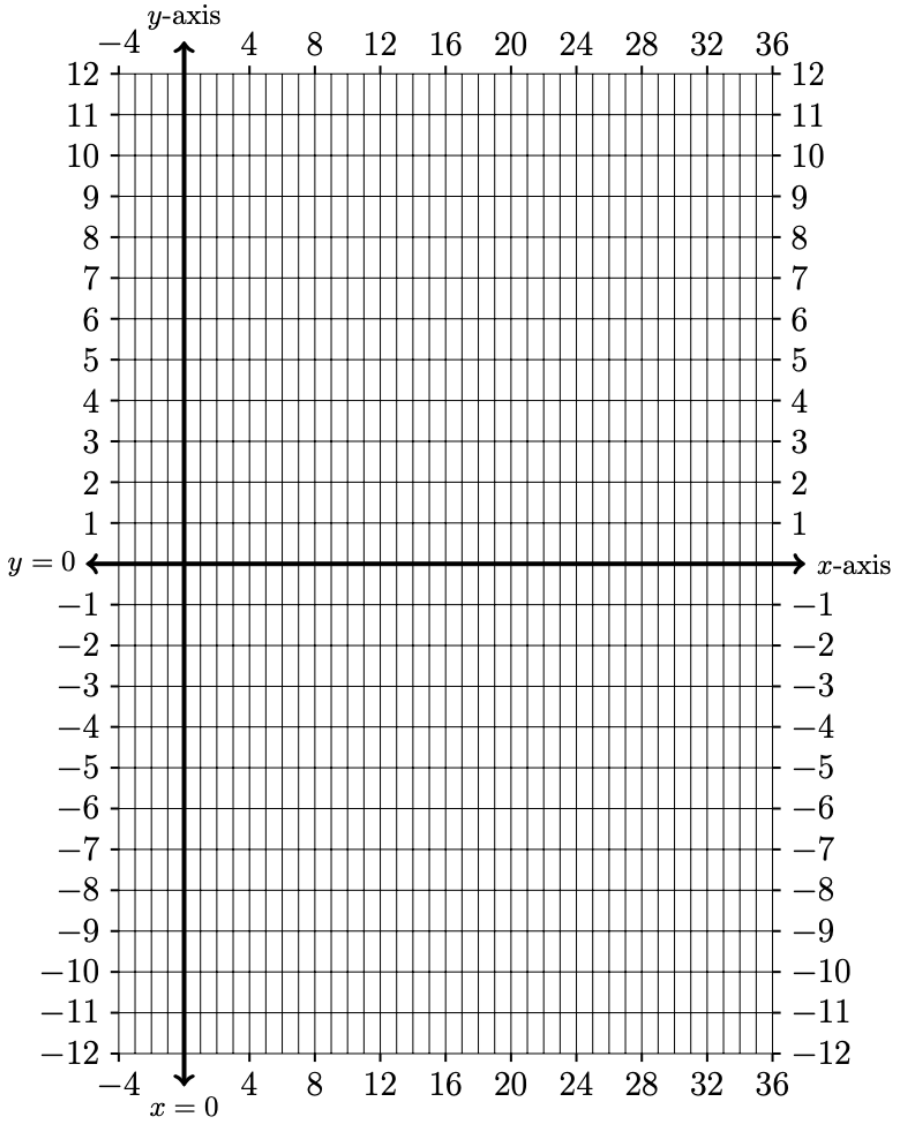
3. VERTICAL SHIFTS OF A ROOT FUNCTION

3A. Let's test the second draft of your conjecture from Problem 2C on a different type of function. Consider the following absolute value functions

$f(x) = \sqrt{x}$, $g(x) = f(x) + 8$, $h(x) = f(x) - 6$

For each function, specifically identify the value of positive constant $c > 0$. Then, create a table of values for $f(x)$. Without creating a table for $g(x)$ or $h(x)$, see if you can graph the resulting root function on the axes below.

Input	*Output*
x	$f(x)$
-1	
0	
1	
4	
9	
16	
25	
36	



3B. Now use a graphing calculator (like Desmos.com or a TI-Calculator) to graph the functions:

$f(x) = \sqrt{x}$, $g(x) = f(x) + 8$, $h(x) = f(x) - 6$

How accurate was your guess in Problem 3A?

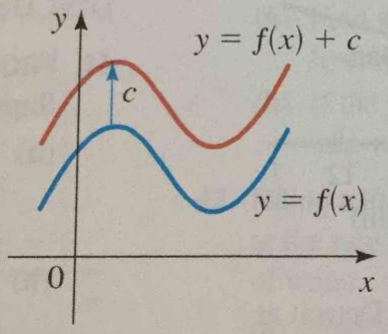
3C. Take a look at the call out box below:

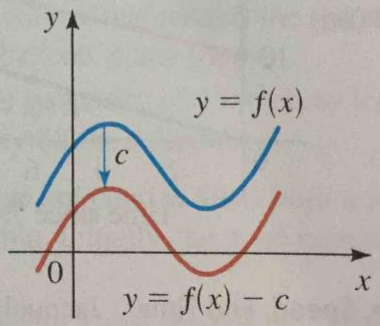
VERTICAL SHIFTS OF GRAPHS

Suppose $c > 0$.

To graph $y = f(x) + c$, shift the graph of $y = f(x)$ upward c units.

To graph $y = f(x) - c$, shift the graph of $y = f(x)$ downward c units.





Translate this into abuelita language for yourself (simple non-technical language that describes what the math is saying so that your abuelita can understand). Really push yourself to make the description simple.

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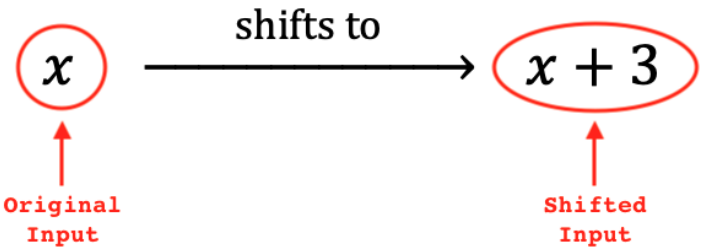
4. HORIZONTAL SHIFTS: SHIFT THE INPUT OF A FUNCTION

4A. Consider the following shifts of the input variable



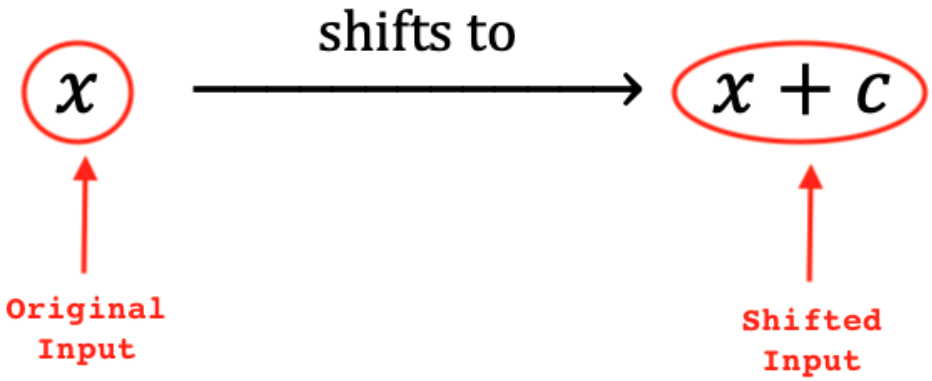
Draw the effect of this shift on the real number line (x −axis) below:

Shifted values:

x−values: −10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10

What do you notice about the zero position in the shifted input $(x + 3)$ versus the zero position in the original input x?

4B. Suppose that $c > 0$. Make a conjecture about the effect of the following shift:



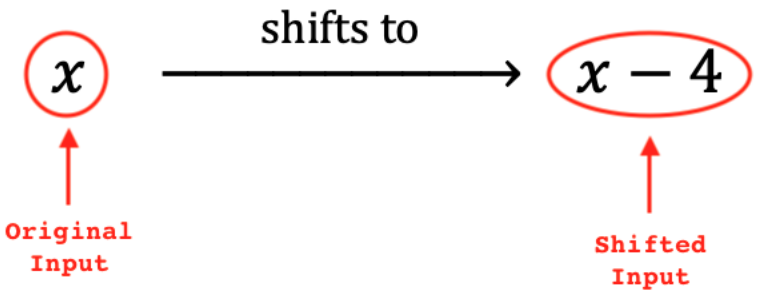
Draw the effect of this shift on the real number line (x −axis) below:

Shifted values:

x−values: −10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10

What does this shift do to the original input? In other words, What do you notice about the zero position in the shifted input $(x + c)$ versus the zero position in the original input x?

4C. Consider the following shifts of the input variable



Draw the effect of this shift on the real number line (x −axis) below:

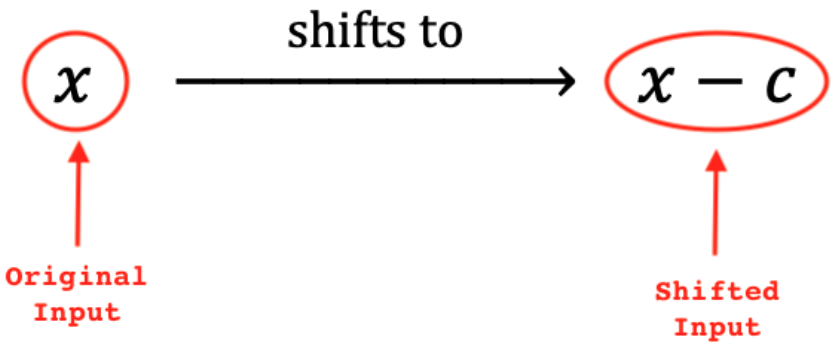
Shifted values:

x−values: −10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10

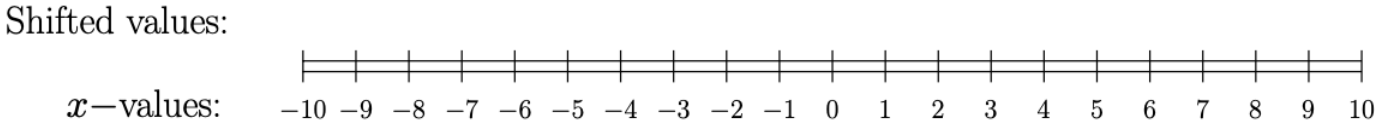
What do you notice about the zero position in the shifted input $(x-4)$ versus the zero position in the original input x?

$$x \xrightarrow{\text{shifts to}} x-c$$

4D. Suppose that $c > 0$. Make a conjecture about the effect of the following shift:



Draw the effect of this shift on the real number line (x −axis) below:



What does this shift do to the original input? In other words, What do you notice about the zero position in the shifted input $(x - c)$ versus the zero position in the original input x?

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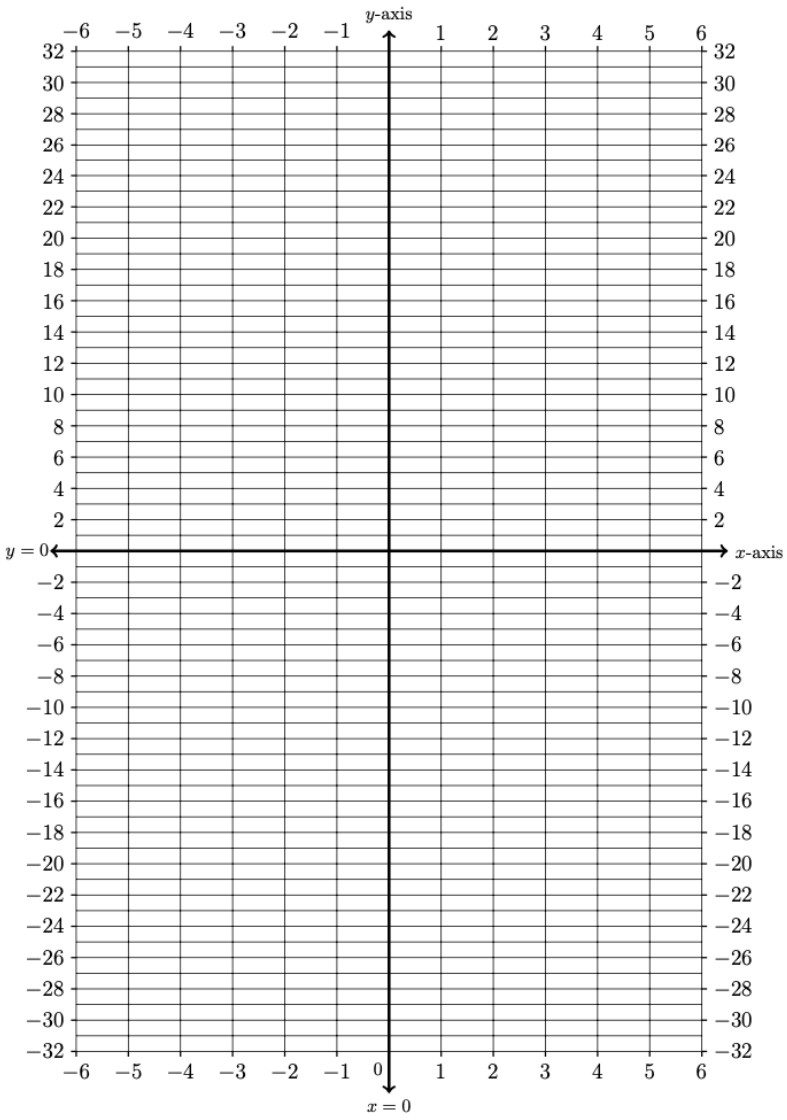
5. HORIZONTAL SHIFTS OF A QUADRATIC FUNCTION

1A. Consider the following quadratic functions

$$f(x) = x^2, \qquad g(x) = f(x+2), \qquad h(x) = f(x-3)$$

Create a table of values and graph the resulting parabolas on these axes below.

Input	*Output*		
x	$f(x)$	$g(x)$	$h(x)$
-6			
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			
5			
6			



5B. Look back at both the graphs and the table of values from Problem 5A. What do you notice about the relationship between the output values of the functions

$$f(x), \qquad g(x) = f(x+2), \quad \text{and} \quad h(x) = f(x-3)$$

5C. Make a conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f(x)$ and a positive constant $c > 0$. Suppose we define functions

$$g(x) = f(x + c) \qquad \text{and} \qquad h(x) = f(x - c)$$

What is the relationship between $f(x)$, $g(x)$, and $h(x)$?

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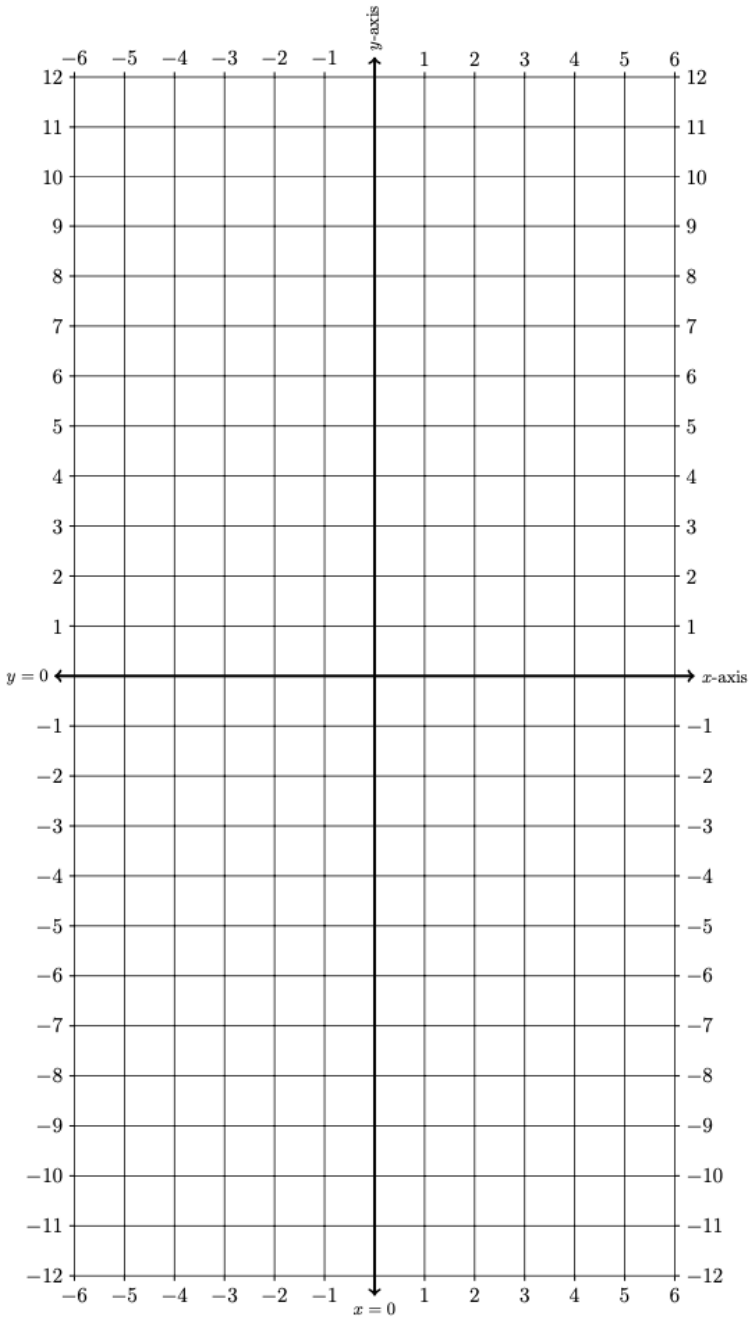
2. HORIZONTAL SHIFTS OF AN ABSOLUTE VALUE FUNCTION

2A. Let's test your conjecture from Problem 1C on a different type of function. Consider the following absolute value functions

$f(x) = |x|,$ $g(x) = f(x+4),$ $h(x) = f(x-3)$

For each function, specifically identify the value of positive constant $c > 0$. Then, create a table of values and graph the resulting curves on these axes below

Input	*Output*		
x	$f(x)$	$g(x)$	$h(x)$
-6			
-5			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			
5			
6			



6B. Look back at both the graphs and the table of values from Problem 6A. What do you notice about the relationship between the output values of the functions

$f(x)$, $g(x) = f(x + 4)$, $h(x) = f(x - 3)$

6C. Revise and update your conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f(x)$ and a positive constant $c > 0$.
Suppose we define functions

$$g(x) = f(x + c) \qquad \text{and} \qquad h(x) = f(x - c)$$

What is the relationship between $f(x)$, $g(x)$, and $h(x)$? Try to put this in both nerdy mathematical language and abuelita language