Math 48A, Lesson 10: Transformations of Functions (Part 1)

1. VERTICAL SHIFTS OF A QUADRATIC FUNCTION

1A. Consider the following quadratic functions

$f\left(x\right)=x^{2}$ , $g\left(x\right)=f\left(x\right)+6$, $h\left(x\right)=f\left(x\right)-10$

Create a table of values and graph the resulting parabolas on these axes below.

|  |  |
| --- | --- |
| *Input* | *Output* |
| $$x$$ | $$f(x)$$ | $$g(x)$$ | $$h(x)$$ |
| $$-6$$ |  |  |  |
| $$-5$$ |  |  |  |
| $$-4$$ |  |  |  |
| $$-3$$ |  |  |  |
| $$-2$$ |  |  |  |
| $$-1$$ |  |  |  |
| $$0$$ |  |  |  |
| $$1$$ |  |  |  |
| $$2$$ |  |  |  |
| $$3$$ |  |  |  |
| $$4$$ |  |  |  |
| $$5$$ |  |  |  |
| $$6$$ |  |  |  |

1B. Look back at both the graphs and the table of values from Problem 1A.

What do you notice about the relationship between the output values of the functions

$f\left(x\right)$, $g\left(x\right)=f\left(x\right)+6$, and $h\left(x\right)=f\left(x\right)-10$

1C. Make a conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f\left(x\right)$ and a positive constant $c>0$.

Suppose we define functions

 $g\left(x\right)=f\left(x\right)+c$ and $h\left(x\right)=f\left(x\right)-c$

What is the relationship between $f\left(x\right)$, $g\left(x\right)$, and $h\left(x\right)$?

2. VERTICAL SHIFTS OF AN ABSOLUTE VALUE FUNCTION

2A. Let’s test your conjecture from Problem 1C on a different type of function. Consider the following absolute value functions

$f\left(x\right)=\left|x\right|$ , $g\left(x\right)=f\left(x\right)+3$, $h\left(x\right)=f\left(x\right)-9$

For each function, specifically identify the value of positive constant $c>0$. Then, create a table of values and graph the resulting curves on these axes below



|  |  |
| --- | --- |
| *Input* | *Output* |
| $$x$$ | $$f(x)$$ | $$g(x)$$ | $$h(x)$$ |
| $$-6$$ |  |  |  |
| $$-5$$ |  |  |  |
| $$-4$$ |  |  |  |
| $$-3$$ |  |  |  |
| $$-2$$ |  |  |  |
| $$-1$$ |  |  |  |
| $$0$$ |  |  |  |
| $$1$$ |  |  |  |
| $$2$$ |  |  |  |
| $$3$$ |  |  |  |
| $$4$$ |  |  |  |
| $$5$$ |  |  |  |
| $$6$$ |  |  |  |

2B. Look back at both the graphs and the table of values from Problem 2A.

What do you notice about the relationship between the output values of the functions

$f\left(x\right)$ , $g\left(x\right)=f\left(x\right)+3$, $h\left(x\right)=f\left(x\right)-10$

2C. Revise and update your conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f\left(x\right)$ and a positive constant $c>0$.

Suppose we define functions

 $g\left(x\right)=f\left(x\right)+c$ and $h\left(x\right)=f\left(x\right)-c$

What is the relationship between $f\left(x\right)$, $g\left(x\right)$, and $h\left(x\right)$? Try to put this in both nerdy mathematical language and abuelita language

3. VERTICAL SHIFTS OF A ROOT FUNCTION

3A. Let’s test the second draft of your conjecture from Problem 2C on a different type of function. Consider the following absolute value functions

$f\left(x\right)=\sqrt{ x }$ , $g\left(x\right)=f\left(x\right)+8$, $h\left(x\right)=f\left(x\right)-6$

For each function, specifically identify the value of positive constant $c>0$. Then, create a table of values for $f\left(x\right)$. Without creating a table for $g\left(x\right)$ or $h\left(x\right)$ , see if you can graph the resulting root function on the axes below.



|  |  |
| --- | --- |
| *Input* | *Output* |
| $$x$$ | $$f(x)$$ |
| $$-1$$ |  |
| $$0$$ |  |
| $$1$$ |  |
| $$4$$ |  |
| $$9$$ |  |
| $$16$$ |  |
| $$25$$ |  |
| $$36$$ |  |

3B. Now use a graphing calculator (like Desmos.com or a TI-Calculator) to graph the functions:

$f\left(x\right)=\sqrt{ x }$ , $g\left(x\right)=f\left(x\right)+8$, $h\left(x\right)=f\left(x\right)-6$

How accurate was your guess in Problem 3A?

3C. Take a look at the call out box below:



Translate this into abuelita language for yourself (simple non-technical language that describes what the math is saying so that your abuelita can understand). Really push yourself to make the description simple.

4. HORIZONTAL SHIFTS: SHIFT THE INPUT OF A FUNCTION

4A. Consider the following shifts of the input variable



 Draw the effect of this shift on the real number line ($x-$axis) below:



What do you notice about the zero position in the shifted input $(x+3)$ versus the zero position in the original input $x$?

4B. Suppose that $c>0$. Make a conjecture about the effect of the following shift:



Draw the effect of this shift on the real number line ($x-$axis) below:



What does this shift do to the original input? In other words, What do you notice about the zero position in the shifted input $(x+c)$ versus the zero position in the original input $x$?

4C. Consider the following shifts of the input variable



 Draw the effect of this shift on the real number line ($x-$axis) below:



What do you notice about the zero position in the shifted input $(x-4)$ versus the zero position in the original input $x$?

$$x → x-c$$

4D. Suppose that $c>0$. Make a conjecture about the effect of the following shift:



Draw the effect of this shift on the real number line ($x-$axis) below:



What does this shift do to the original input? In other words, What do you notice about the zero position in the shifted input $(x-c)$ versus the zero position in the original input $x$?

5. HORIZONTAL SHIFTS OF A QUADRATIC FUNCTION

1A. Consider the following quadratic functions

$f\left(x\right)=x^{2}$ , $g\left(x\right)=f\left(x+2\right)$, $h\left(x\right)=f\left(x-3\right)$

Create a table of values and graph the resulting parabolas on these axes below.

|  |  |
| --- | --- |
| *Input* | *Output* |
| $$x$$ | $$f(x)$$ | $$g(x)$$ | $$h(x)$$ |
| $$-6$$ |  |  |  |
| $$-5$$ |  |  |  |
| $$-4$$ |  |  |  |
| $$-3$$ |  |  |  |
| $$-2$$ |  |  |  |
| $$-1$$ |  |  |  |
| $$0$$ |  |  |  |
| $$1$$ |  |  |  |
| $$2$$ |  |  |  |
| $$3$$ |  |  |  |
| $$4$$ |  |  |  |
| $$5$$ |  |  |  |
| $$6$$ |  |  |  |

5B. Look back at both the graphs and the table of values from Problem 5A.

What do you notice about the relationship between the output values of the functions

$f\left(x\right)$, $g\left(x\right)=f\left(x+2\right)$, and $h\left(x\right)=f\left(x-3\right)$

5C. Make a conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f\left(x\right)$ and a positive constant $c>0$.

Suppose we define functions

 $g\left(x\right)=f\left(x+c\right)$ and $h\left(x\right)=f\left(x-c\right)$

What is the relationship between $f\left(x\right)$, $g\left(x\right)$, and $h\left(x\right)$?

2. HORIZONTAL SHIFTS OF AN ABSOLUTE VALUE FUNCTION

2A. Let’s test your conjecture from Problem 1C on a different type of function. Consider the following absolute value functions

$f\left(x\right)=\left|x\right|$ , $g\left(x\right)=f\left(x+4\right)$, $h\left(x\right)=f\left(x-3\right)$

For each function, specifically identify the value of positive constant $c>0$. Then, create a table of values and graph the resulting curves on these axes below



|  |  |
| --- | --- |
| *Input* | *Output* |
| $$x$$ | $$f(x)$$ | $$g(x)$$ | $$h(x)$$ |
| $$-6$$ |  |  |  |
| $$-5$$ |  |  |  |
| $$-4$$ |  |  |  |
| $$-3$$ |  |  |  |
| $$-2$$ |  |  |  |
| $$-1$$ |  |  |  |
| $$0$$ |  |  |  |
| $$1$$ |  |  |  |
| $$2$$ |  |  |  |
| $$3$$ |  |  |  |
| $$4$$ |  |  |  |
| $$5$$ |  |  |  |
| $$6$$ |  |  |  |

6B. Look back at both the graphs and the table of values from Problem 6A.

What do you notice about the relationship between the output values of the functions

$f\left(x\right)$ , $g\left(x\right)=f\left(x+4\right)$, $h\left(x\right)=f\left(x-3\right)$

6C. Revise and update your conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function $f\left(x\right)$ and a positive constant $c>0$.

Suppose we define functions

 $g\left(x\right)=f\left(x+c\right)$ and $h\left(x\right)=f\left(x-c\right)$

What is the relationship between $f\left(x\right)$, $g\left(x\right)$, and $h\left(x\right)$? Try to put this in both nerdy mathematical language and abuelita language