Math 48A, Lesson 10: Transformations of Functions (Part 1)

1. VERTICAL SHIFTS OF A QUADRATIC FUNCTION

1A. Consider the following quadratic functions

, ,

Table

Description automatically generatedCreate a table of values and graph the resulting parabolas on these axes below.

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| *Input* | *Output* | | |
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1B. Look back at both the graphs and the table of values from Problem 1A.

What do you notice about the relationship between the output values of the functions

, , and

1C. Make a conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function and a positive constant .

Suppose we define functions

and

What is the relationship between , , and ?

2. VERTICAL SHIFTS OF AN ABSOLUTE VALUE FUNCTION

2A. Let’s test your conjecture from Problem 1C on a different type of function. Consider the following absolute value functions

, ,

For each function, specifically identify the value of positive constant . Then, create a table of values and graph the resulting curves on these axes below

A picture containing table

Description automatically generated

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2B. Look back at both the graphs and the table of values from Problem 2A.

What do you notice about the relationship between the output values of the functions

, ,

2C. Revise and update your conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function and a positive constant .

Suppose we define functions

and

What is the relationship between , , and ? Try to put this in both nerdy mathematical language and abuelita language

3. VERTICAL SHIFTS OF A ROOT FUNCTION

3A. Let’s test the second draft of your conjecture from Problem 2C on a different type of function. Consider the following absolute value functions

, ,

For each function, specifically identify the value of positive constant . Then, create a table of values for . Without creating a table for or , see if you can graph the resulting root function on the axes below.

Chart

Description automatically generated

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3B. Now use a graphing calculator (like Desmos.com or a TI-Calculator) to graph the functions:

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How accurate was your guess in Problem 3A?

3C. Take a look at the call out box below:

A picture containing text, whiteboard

Description automatically generated

Translate this into abuelita language for yourself (simple non-technical language that describes what the math is saying so that your abuelita can understand). Really push yourself to make the description simple.

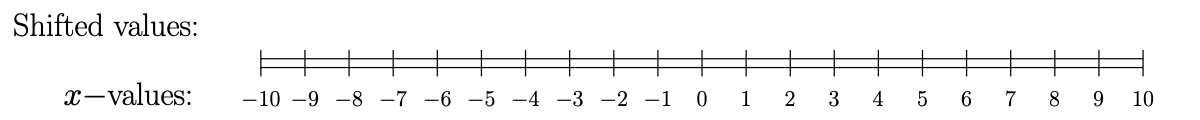
4. HORIZONTAL SHIFTS: SHIFT THE INPUT OF A FUNCTION

4A. Consider the following shifts of the input variable

Diagram

Description automatically generated with medium confidence

Draw the effect of this shift on the real number line (axis) below:



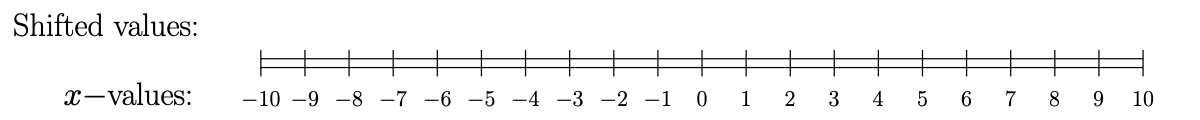
What do you notice about the zero position in the shifted input versus the zero position in the original input ?

4B. Suppose that . Make a conjecture about the effect of the following shift:

Diagram

Description automatically generated

Draw the effect of this shift on the real number line (axis) below:



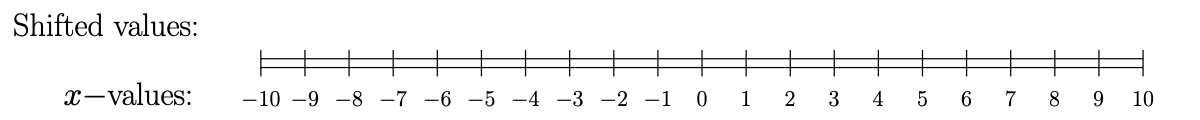
What does this shift do to the original input? In other words, What do you notice about the zero position in the shifted input versus the zero position in the original input ?

4C. Consider the following shifts of the input variable

Diagram

Description automatically generated

Draw the effect of this shift on the real number line (axis) below:



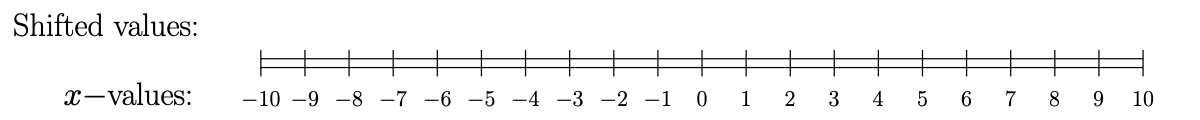
What do you notice about the zero position in the shifted input versus the zero position in the original input ?

4D. Suppose that . Make a conjecture about the effect of the following shift:

Diagram

Description automatically generated

Draw the effect of this shift on the real number line (axis) below:



What does this shift do to the original input? In other words, What do you notice about the zero position in the shifted input versus the zero position in the original input ?

5. HORIZONTAL SHIFTS OF A QUADRATIC FUNCTION

1A. Consider the following quadratic functions

, ,

Table

Description automatically generatedCreate a table of values and graph the resulting parabolas on these axes below.

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| *Input* | *Output* | | |
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5B. Look back at both the graphs and the table of values from Problem 5A.

What do you notice about the relationship between the output values of the functions

, , and

5C. Make a conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function and a positive constant .

Suppose we define functions

and

What is the relationship between , , and ?

2. HORIZONTAL SHIFTS OF AN ABSOLUTE VALUE FUNCTION

2A. Let’s test your conjecture from Problem 1C on a different type of function. Consider the following absolute value functions

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For each function, specifically identify the value of positive constant . Then, create a table of values and graph the resulting curves on these axes below

A picture containing table

Description automatically generated

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6B. Look back at both the graphs and the table of values from Problem 6A.

What do you notice about the relationship between the output values of the functions

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6C. Revise and update your conjecture (a mathematical guess) about what happens in the following scenario:

Assume we have a function and a positive constant .

Suppose we define functions

and

What is the relationship between , , and ? Try to put this in both nerdy mathematical language and abuelita language