
Free Response: Solve each of the following problems. Show your work and box your final answer.

$\boxed{3}$ 1. Solve the following rational equation. Show your work:

$$x - \frac{2x}{x+3} = \frac{6}{x+3}$$

Solution:

$$x - \frac{2x}{x+3} = \frac{6}{x+3}$$

$$\Longrightarrow \quad x \cdot \frac{(x+3)}{(x+3)} - \frac{2x}{x+3} = \frac{6}{x+3}$$

$$\Longrightarrow \quad x^2 + 3x - 2x = 6$$

$$\Longrightarrow \quad x^2 + x - 6 = 0$$

$$\Longrightarrow \quad (x+3) \cdot (x-2) = 0$$

$$\Longrightarrow \quad \boxed{x = 2} \text{ and } \boxed{x \neq -3}$$

$\boxed{3}$ 2. Simplify using rules of radicals: $\sqrt[3]{n^4}$

Solution:

$$\begin{aligned} \sqrt[3]{n^4} &= \sqrt[3]{(n \cdot n \cdot n) \cdot n} \\ &= \sqrt[3]{n^3 \cdot n} \\ &= \sqrt[3]{n^3} \cdot \sqrt[3]{n} \\ &= \boxed{n \cdot \sqrt[3]{n}} \end{aligned}$$

$\boxed{3}$ 3. Simplify each expression below as much as possible. Show your work.

$$\frac{2t}{t^2-1}+\frac{-1}{t-1}$$

Solution:

$$\begin{aligned}
&= \frac{2t}{(t+1)\cdot(t-1)}+\frac{-1}{t-1}\cdot\frac{(t+1)}{(t+1)} \\
&= \frac{2t-(t+1)}{(t+1)\cdot(t-1)} \\
&= \frac{t-1}{(t+1)\cdot(t-1)} \\
&= \boxed{\frac{1}{t+1}}
\end{aligned}$$

- Recall that $\dfrac{t-1}{t-1}=1$ if $t\neq 1$.

$\boxed{3}$ 4. In your own words, explain the inverse operation for rational equations. Then, explain how to use this inverse to rational equations (Hint: see problem 3 above.)

Solution:

- We've studied two inverse operations for rational equations:

    1. If $\dfrac{A}{B}=D$, then $A=BD$.
    2. If $\dfrac{A}{D}=\dfrac{B}{D}$, then $A=B$.

- We can describe inverse 1 in English as follows: if we have a rational equation and one side of the equation has denominator B, we can eliminate this denominator by multiplying both sides of the equation by B.
- We can describe inverse 2 in English as follows: if we have a rational equation where equal fractions have identical denominators, then we eliminate these denominators and set the numerators equal to each other.
- In order to solve rational equation using either of these inverses, we manipulate the expressions on each side of the equals sign to get the entire expression combined into one fraction with a single denominator. Then, depending on which inverse is applicable, we annihilate denominators using the appropriate technique and solve the equation that results.

$\boxed{3}$ 5. Simplify the following expression. Show your work. $\sqrt[8]{a^{11}} \cdot \sqrt[8]{a^5}$

Solution:

$$\begin{aligned}
\sqrt[8]{a^{11}} \cdot \sqrt[8]{a^5} &= \sqrt[8]{a^{11} \cdot a^5} \\
&= \sqrt[8]{a^{11+5}} \\
&= \sqrt[8]{a^{16}} \\
&= a^{16/8} \\
&= \boxed{a^2}
\end{aligned}$$

$\boxed{3}$ 6. Simplify the following expression. Show your work. $\dfrac{\sqrt{12w^7y}}{4\sqrt{3w^3y^4}}$

Solution:

$$\begin{aligned}
\frac{\sqrt{12 \cdot w^7 \cdot y}}{4 \cdot \sqrt{3 \cdot w^3 \cdot y^4}} &= \frac{1}{4} \cdot \frac{\sqrt{12 \cdot w^7 \cdot y}}{\sqrt{3 \cdot w^3 \cdot y^4}} \\
&= \frac{1}{4} \cdot \sqrt{\frac{12}{3} \cdot \frac{w^7}{w^3} \cdot \frac{y}{y^4}} \\
&= \frac{1}{4} \cdot \sqrt{\frac{4 \cdot w^4}{y^3}} \\
&= \frac{1}{4} \cdot \frac{\sqrt{4} \cdot \sqrt{w^4}}{\sqrt{y^3}} \\
&= \boxed{\frac{w^2}{2 \cdot |y| \cdot \sqrt{y}}}
\end{aligned}$$

7. Solve the following equation. Show your work.

$$3 + \sqrt[3]{y-3} = 5$$

Solution:

$$\implies \qquad \sqrt[3]{y-3} = 2$$

$$\implies \qquad \left(\sqrt[3]{y-3}\right)^3 = 2^3$$

$$\implies \qquad y - 3 = 8$$

$$\implies \qquad \boxed{y = 11}$$

$\boxed{2}$ 8. In your own words, explain the inverse operations for power expressions:

$$\sqrt[2]{\ x^2\ } = |x|, \qquad \sqrt[3]{\ x^3\ } = x$$

Explain why we use an absolute value to take the inverse of x^2 using a square root. Explain why no absolute value sign is necessary when we take the inverse of x^3 using a cube root. (Hint: see problem 8.)

Solution:

- Recall that radicals are the inverse of power operations, with

$$b = \sqrt[n]{a} \qquad \implies \qquad b^n = a$$

- With this in mind, let's take a look at an example of a square root problem from our table above:

$$b = \sqrt[2]{4} \qquad \implies \qquad b^2 = 4$$

  We are searching for a number b such that when we square b, we get $+4$. From our knowledge of arithmetic, there are two possible candidates that satisfy this equation, which are either $b = -2$ or $b = +2$. We agree that when dealing with square roots, we will always choose the positive number, so that $\sqrt[2]{4} = +2$. This agreement is a convention for all square root problems. When we consider $\sqrt[2]{x^2}$, very special behavior is happening when $x < 0$. Because $x \cdot x > 0$ (two negatives multiplied together make a positive), the $\sqrt[2]{x^2} = |x|$. In other words, the square root of a variable squared turned negative input values x into the positive versions while leaving positive values of x as positives.

- On the other hand, let's take a look at an example of a cube root problem from our table above:

$$b = \sqrt[3]{-8} \qquad \implies \qquad b^3 = -8$$

  In this case, there is only one possible choice for b which is $b = -2$. When taking the cube root, there is no ambiguity in choosing which number we should use. The clarity we experience here is a direct result of the special behavior of x^3. Because we multiply x by itself three times, the power x^3 is positive when x is positive and negative when x is negative. Thus, no choice needs to be made when we invert this operation.